AD710725

August 1970
Report 1341-26F

APPLICATIONS OF CUMULATIVE DAMAGE IN THE PREPARATION OF PARAMETRIC GRAIN DESIGN CURVES AND THE PREDICTION OF GRAIN FAILURES ON PRESSURIZATION

FINAL REPORT

1 April 1969 through 28 February 1970

VOLUME II - APPENDICES A THROUGH M

By

K. W. Bills, Jr., D. M. Campbell, R. D. Steele and
J. D. McConnell

Aerojet Solid Propulsion Company
Sacramento, California

and

Consultants: L. R. Herrmann, University of California and
R. J. Farris, University of Utah

Prepared For

Department of the Navy
Naval Ordnance Systems Command (ORD-0331)
Contract No. N00017-69-C-4423

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APPLICATIONS OF CUMULATIVE DAMAGE IN THE PREPARATION OF PARAMETRIC DESIGN CURVES AND THE PREDICTION OF GRAIN FAILURES ON PRESSURIZATION

VOLUME II - APPENDICES A THRU M

PREPARED FOR

DEPARTMENT OF THE NAVY
NAVAL ORDNANCE SYSTEMS COMMAND (ORD-0331)
CONTRACT NO. N00017-69-C-4423

Prepared by:
K. W. Bills, Jr.
Associate Scientist
Propellant Physics

Approved by:
G. J. Svob, Manager
Integrated Grain Design
and Weights, Department 4320

Aerojet Solid Propulsion Company

Report 1341-26F

VOLUME II

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APPENDICES

APPENDIX A

MODULUS DATA INPUT FOR THE COMPUTER

Appendix A

MODULUS DATA INPUT FOR THE COMPUTER

The relaxation modulus was represented by a sixteen-term Prony series. This relation for the tensile modulus, E(t), is

$$E(t) = A_o + \sum_{i=1}^{m} A_i e^{-\beta_i t} \qquad \text{(A-1)}$$

where A_o, A_i, and β_i are constants.

For an incompressible material the relaxation modulus in shear, $\mu(t)$, is given by

$$\mu(t) = \frac{E(t)}{3} = \frac{A_c}{3} + \sum_{i=1}^{m} \frac{A_i}{3} e^{-\beta_i t} \qquad \text{(A-2)}$$

The viscoelastic stress analyses requires the shear modulus representation.

The Prony series constants employed in the present parameter study are given in Table A-1, for the CTPB propellant, and in Table A-2, for the HTPB propellant.

To complete the viscoelastic description the time-temperature shift function, a_T, is required. Hence, also listed in Tables A-1 and A-2 are the logarithims of a_T at 12 different temperatures from -100° to +200°F, for the two referenced propellants.

TABLE A-2

MODULUS INPUT FOR THE HTPB PROPELLANT

Prony Series Parameters

Log (t/a_T) (min)	E psi	β_i (hr^{-1})	A_i	$A_i/3$	i
			80	26.67	0
-8	10000	3×10^9	4376.65	1458.9	1
-7	7000	3×10^8	3697.78	1232.6	2
-6	4000	3×10^7	2056.49	685.50	3
-5	2400	3×10^6	896.865	298.96	4
-4	1600	3×10^5	665.701	221.90	5
-3	1050	3×10^4	384.337	128.11	6
-2	720	3×10^3	250.028	83.343	7
-1	500	3×10^2	180.589	60.196	8
0	350	3×10^1	106.550	35.517	9
1	260	3×10^0	67.0877	22.363	10
2	205	3×10^{-1}	35.8845	11.962	11
3	170	3×10^{-2}	39.3243	11.441	12
4	140	3×10^{-3}	24.4644	8.155	13
5	120	3×10^{-4}	15.1300	5.043	14
6	110	3×10^{-5}	-2.4636	-0.8212	15
7	100	3×10^{-6}	33.0018	11.001	16

Time-Temperature Shift Function

Temp., °F	$\text{Log}_{10} a_T$
-75	6.90
-50	4.93
-25	3.41
0	2.35
25	1.48
50	0.72
77	0
100	-0.46
125	-0.90
150	-1.24
170	-1.47

APPENDIX B

PARAMETER STUDY FOR HISTORY 1

APPENDIX B

PARAMETER STUDY FOR HISTORY 1

The results of a series of one-dimensional, thermoviscoelastic analyses are presented graphically in this appendix. All of the analyses were based upon the environmental temperature history described in the text and shown separately in each figure. The data for the CTPB propellant are presented in Figures B-1 to B-6, while those for the HTPB propellant are given in Figures B-7 to B-12.

No analyses of the results are made here.

A. CTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 1
(B = 4 in.)

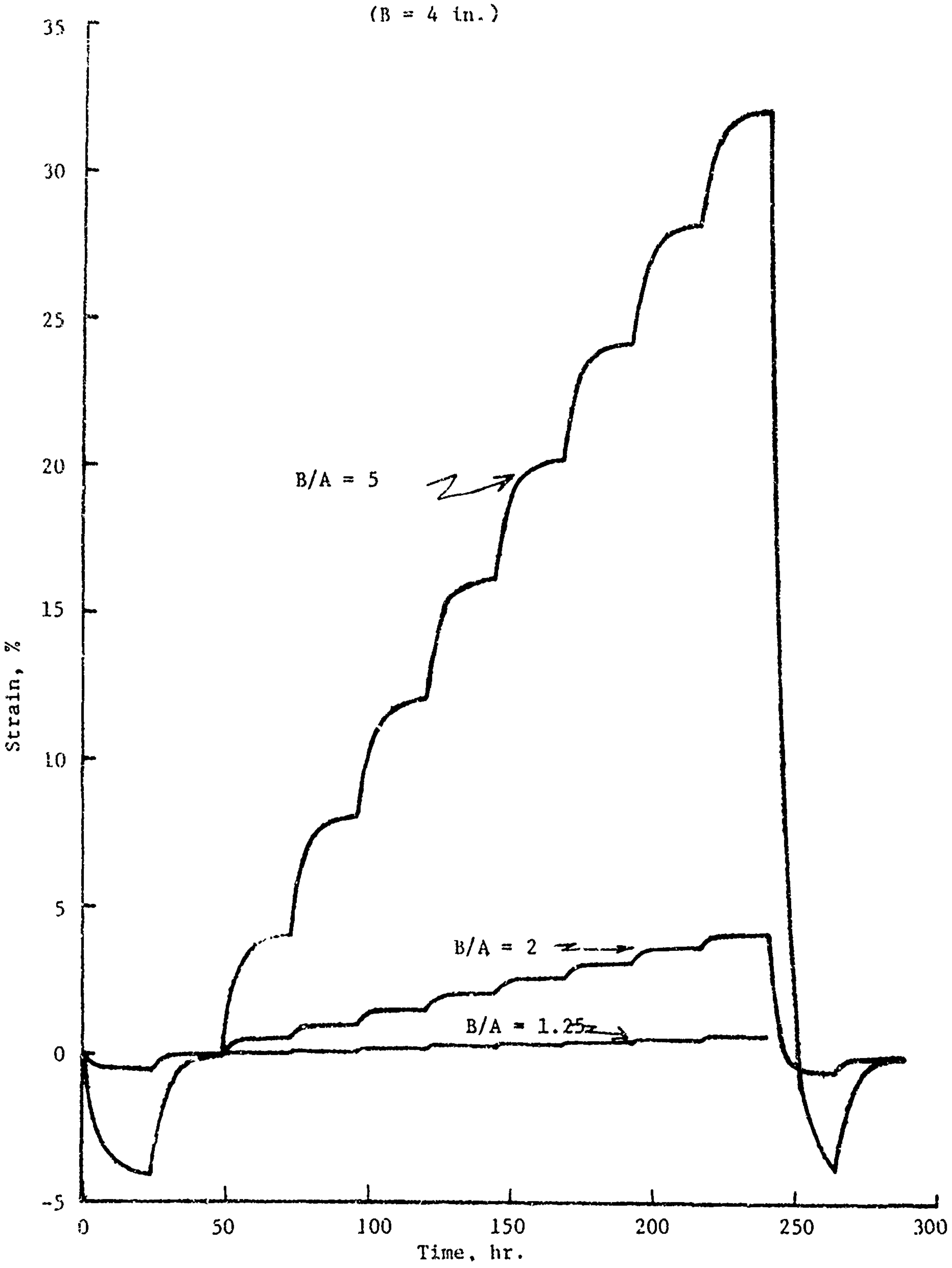

Figure B-1

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 1
(B = 8 in.)

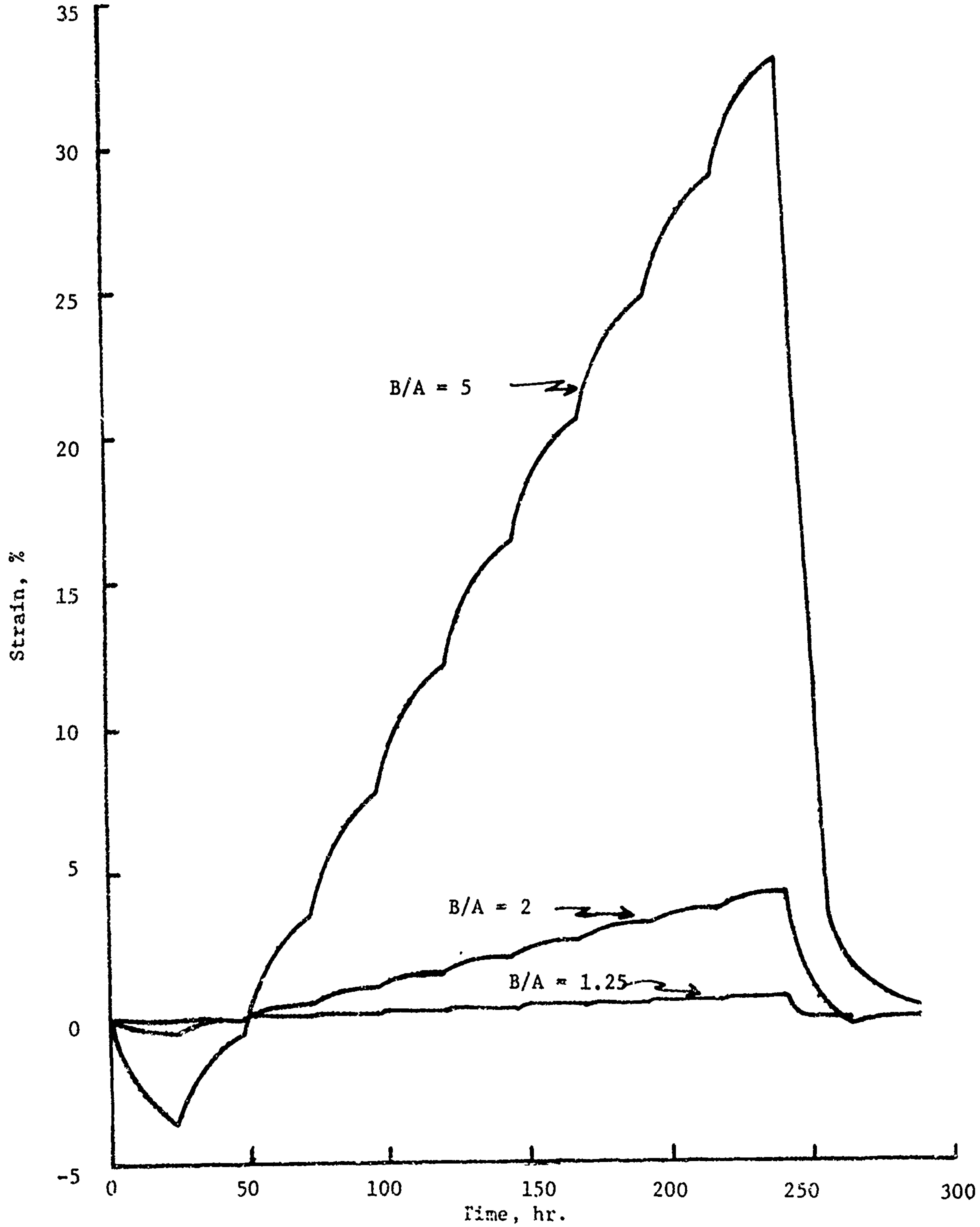

Figure B-2

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1
(B = 4 in.)

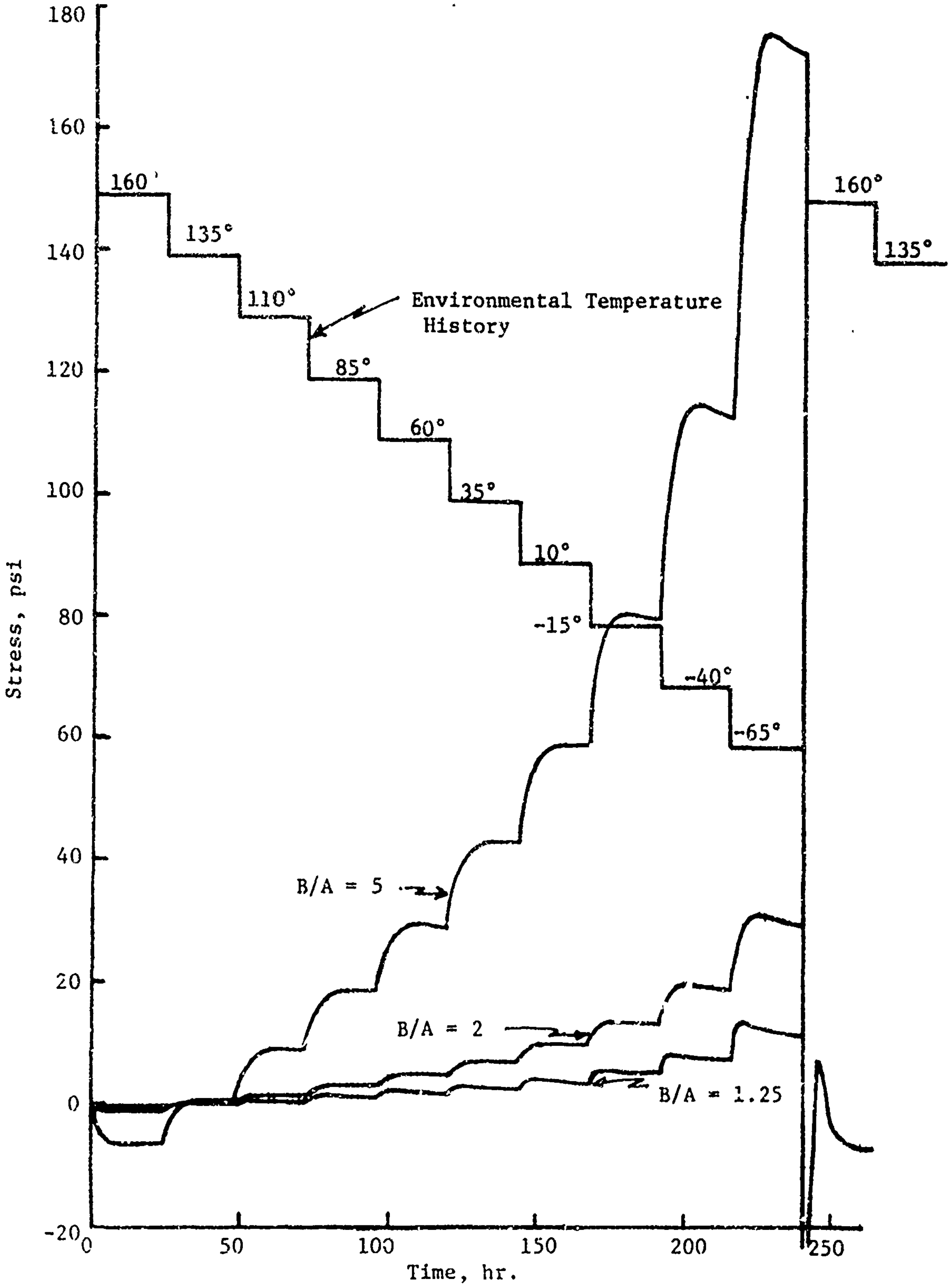

Figure B-3

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1
(B = 8 in.)

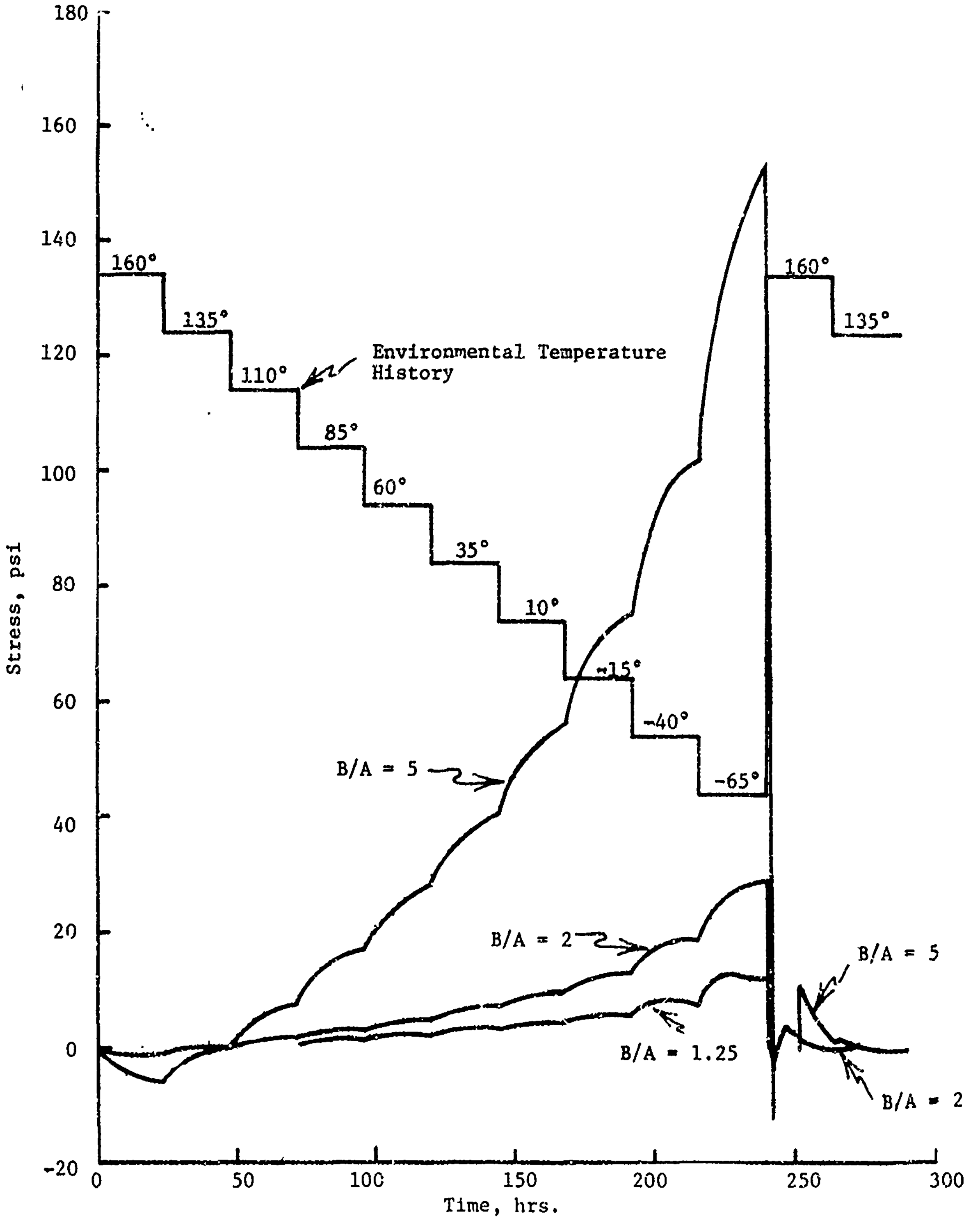

Figure B-4

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1

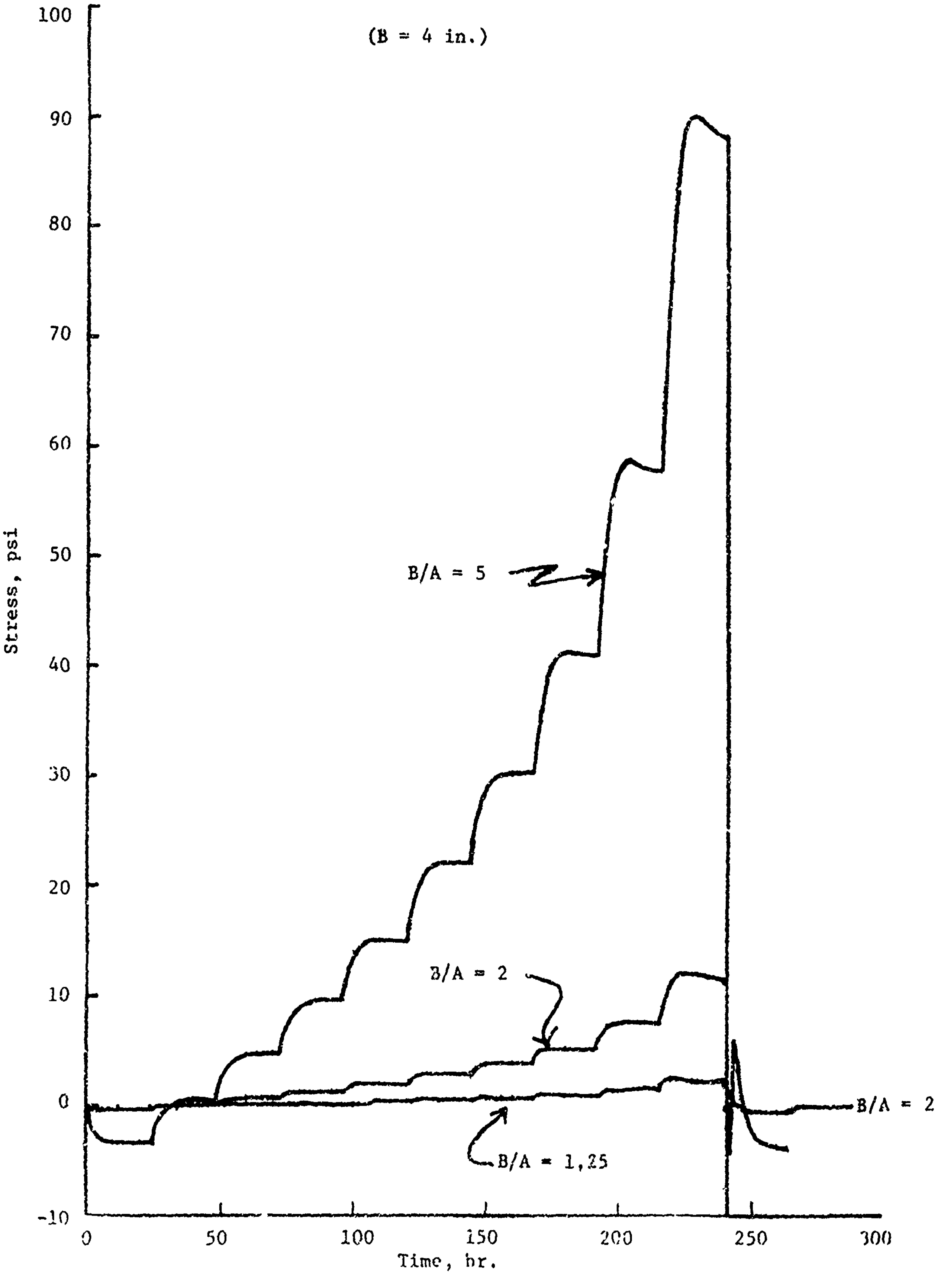

Figure B-5

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1

(B = 8 in.)

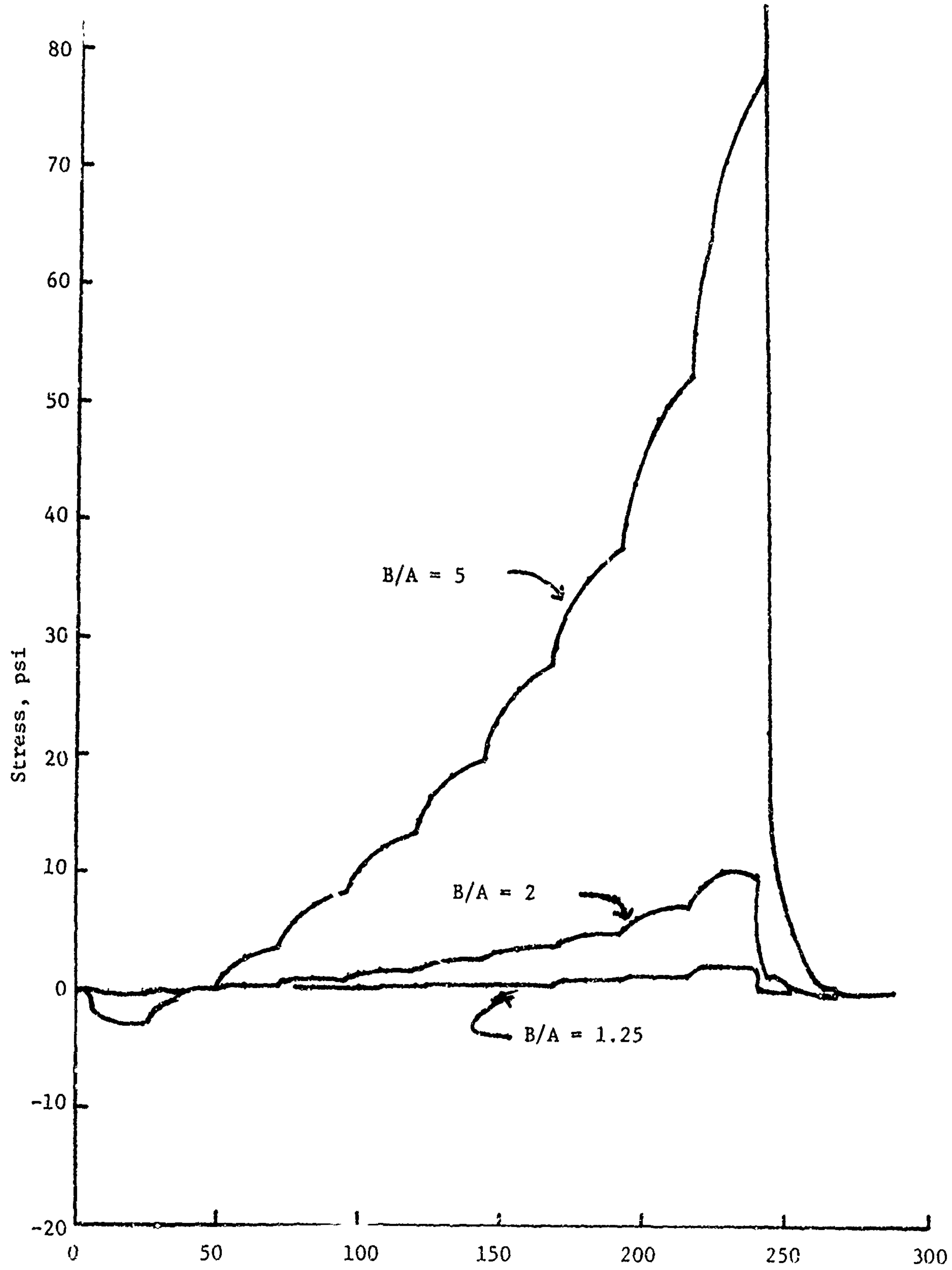

Figure B-6

B. HTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 1

(B = 4 in.)

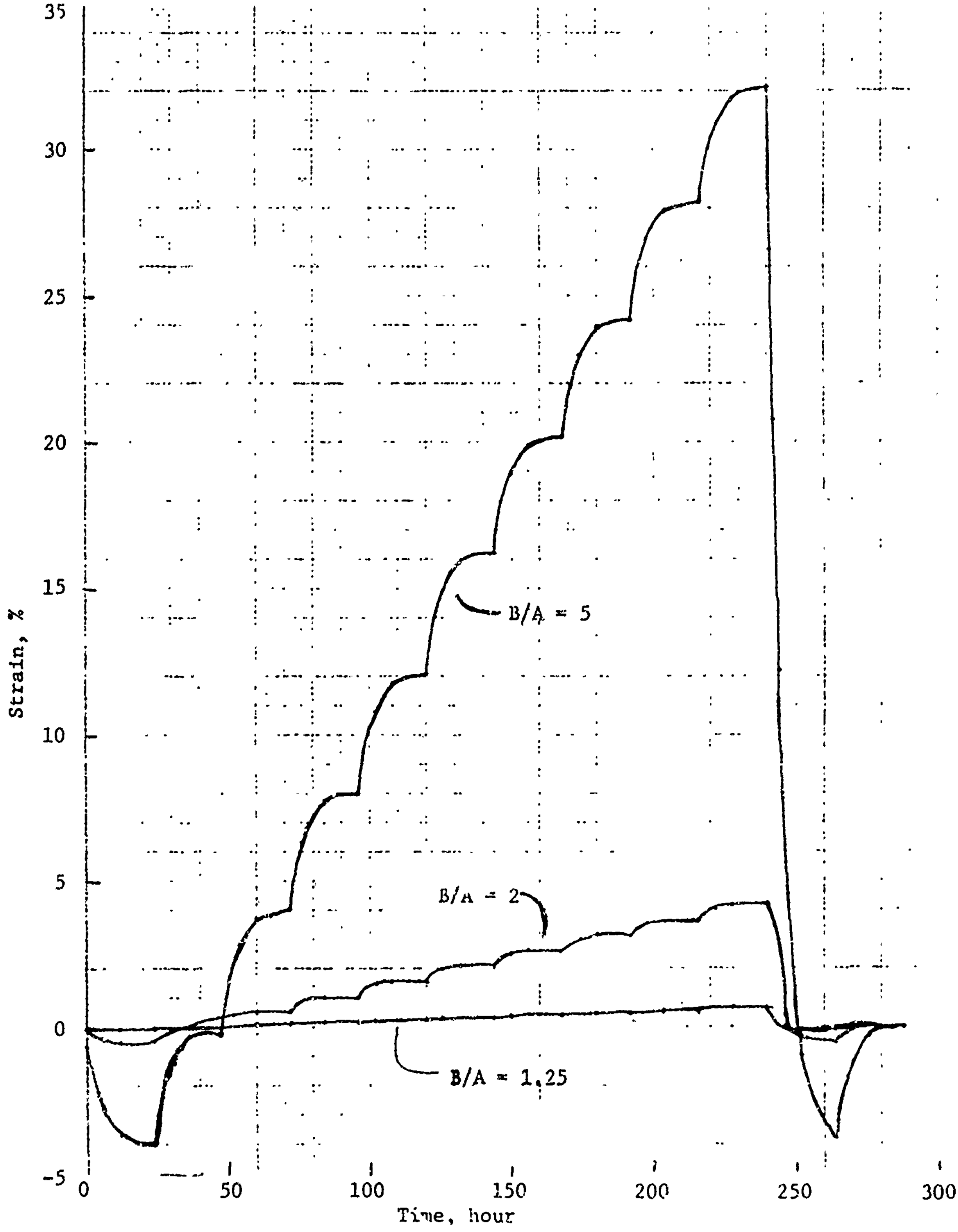

Figure B-7

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 1

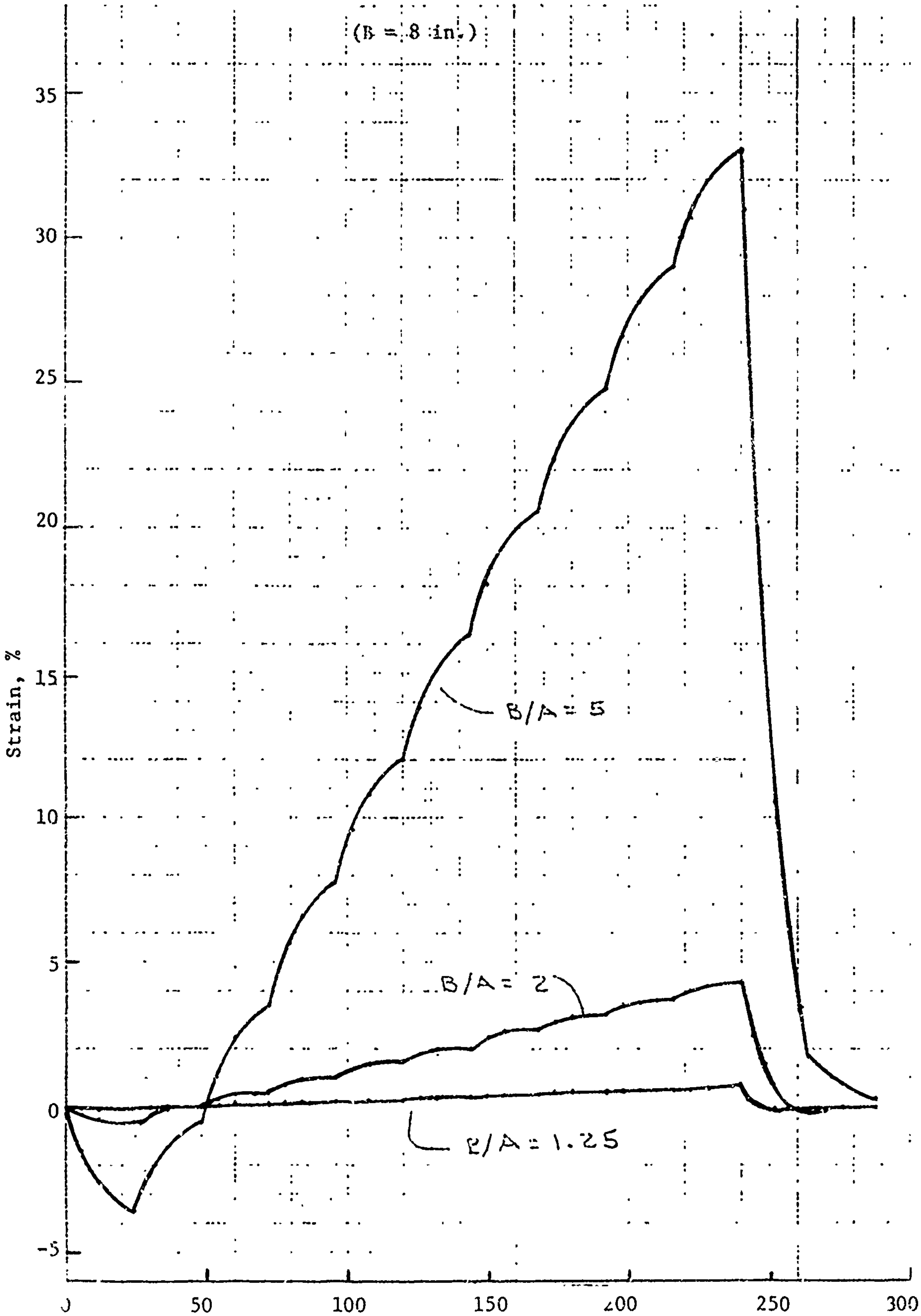

Figure B-8

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1

(B = 4 in.)

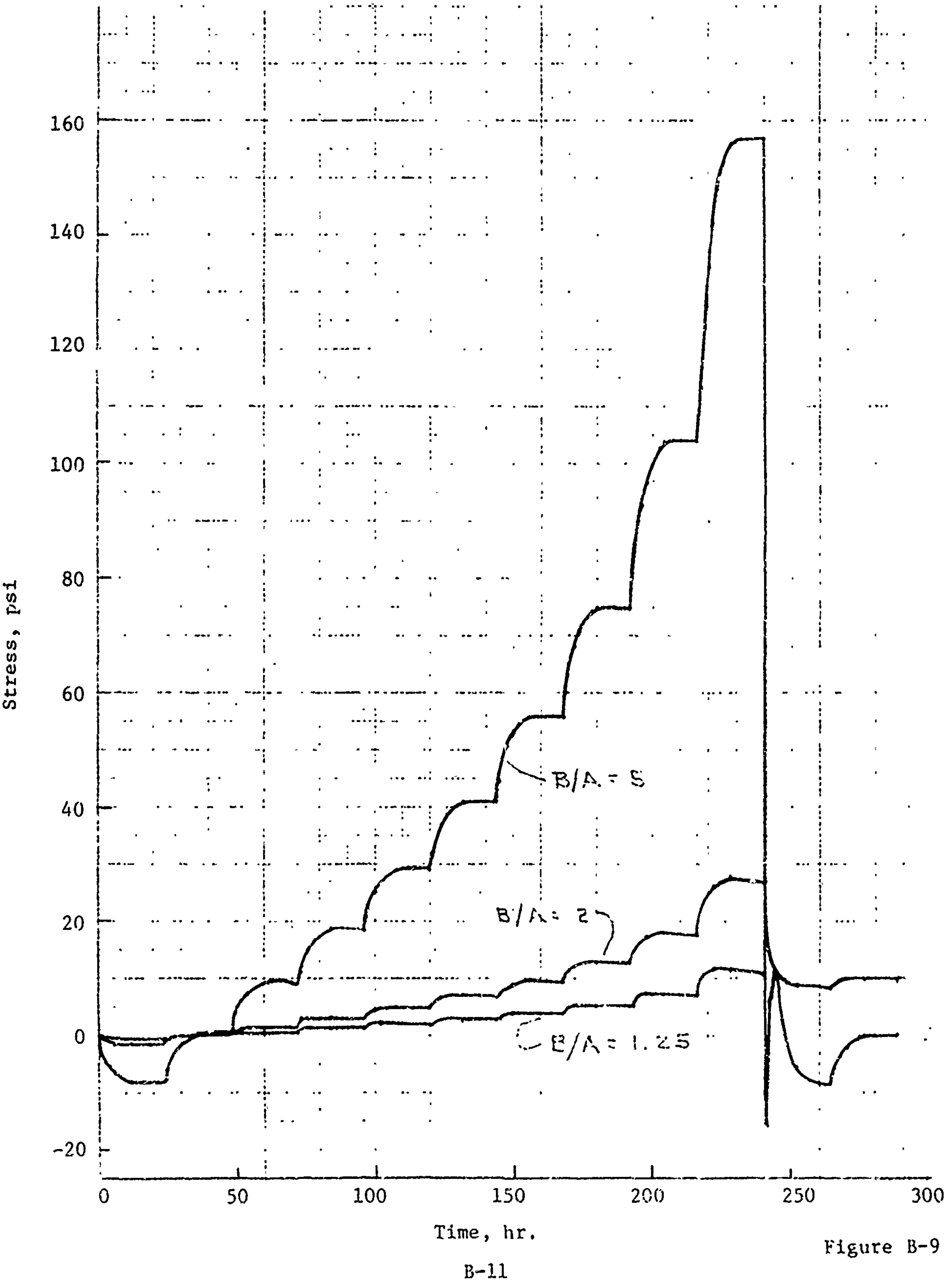

Figure B-9

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1
(B = 8 in.)

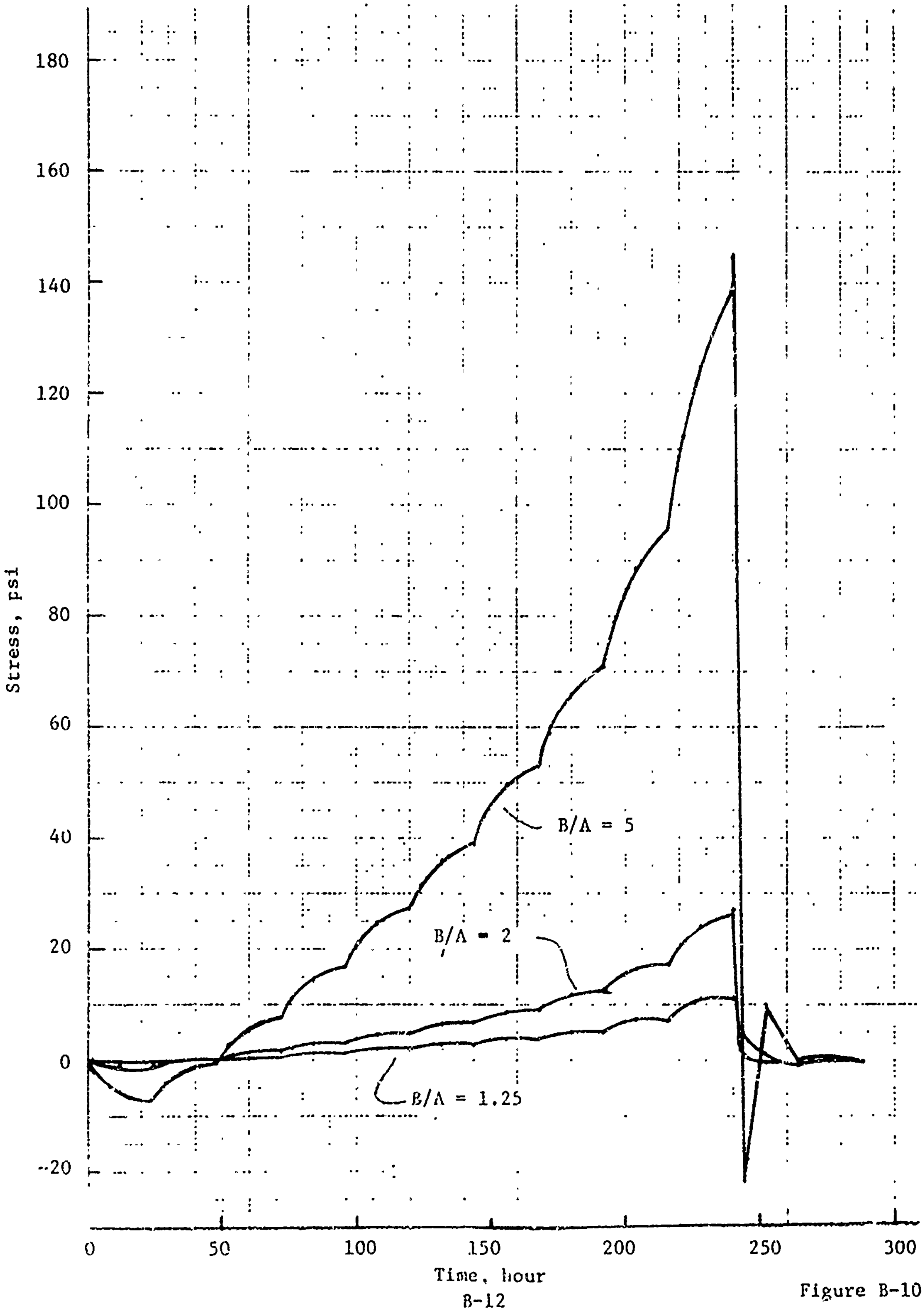

Figure B-10

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1
(B = 4 in.)

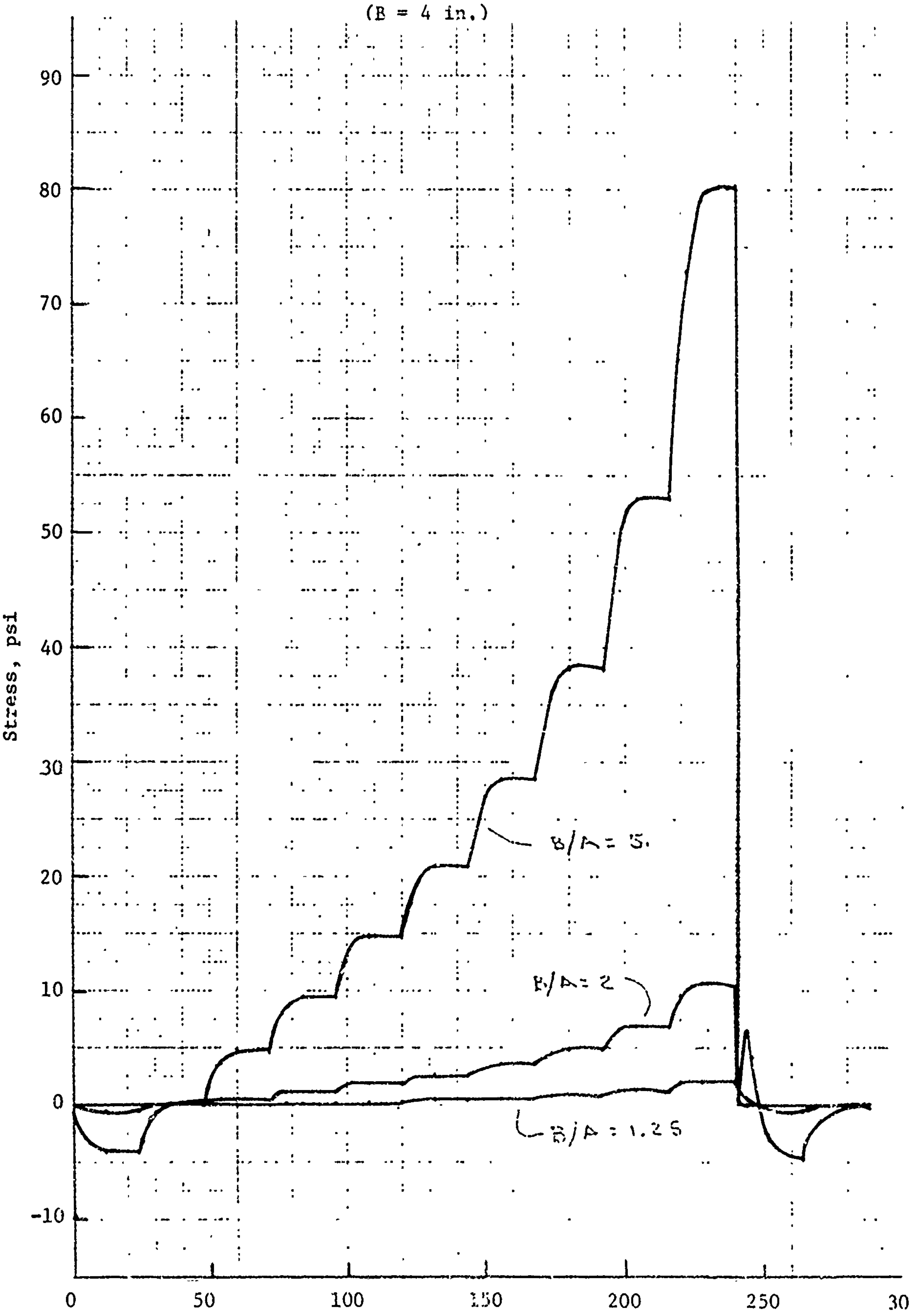

Figure B-11

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1
(B = 8 in.)

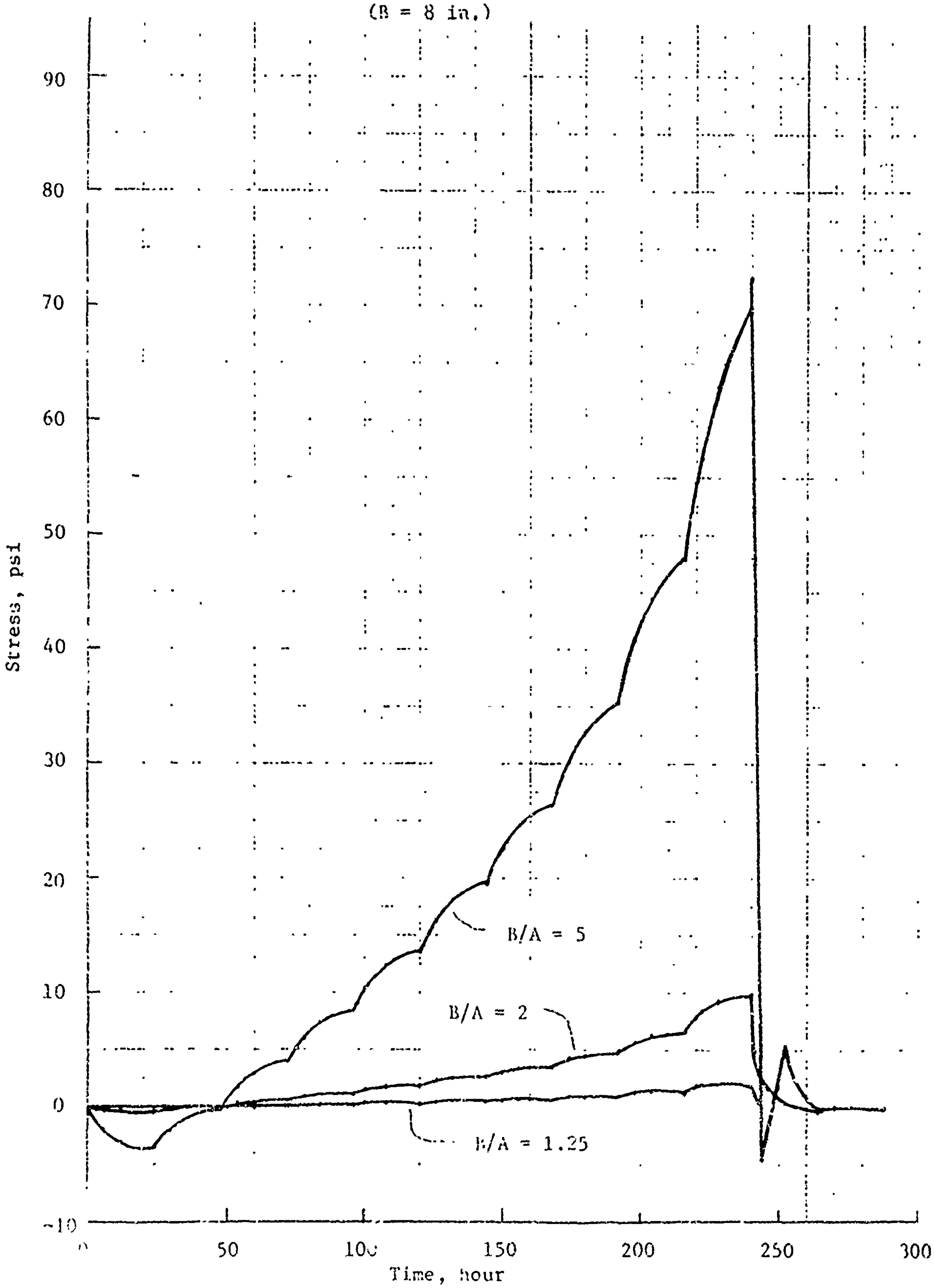

Figure B-12

APPENDIX C

PARAMETER STUDY FOR HISTORY 2

APPENDIX C

PARAMETER STUDY FOR HISTORY 2

The results of a series of one-dimensional, thermoviscoelastic analyses are presented graphically in this appendix. All of the results were based upon the environmental temperature history described in the e text and shown separately in each figure. The data for the CTPB and HTPB propellants are presented separately.

No analyses of the results are made here.

A. CTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

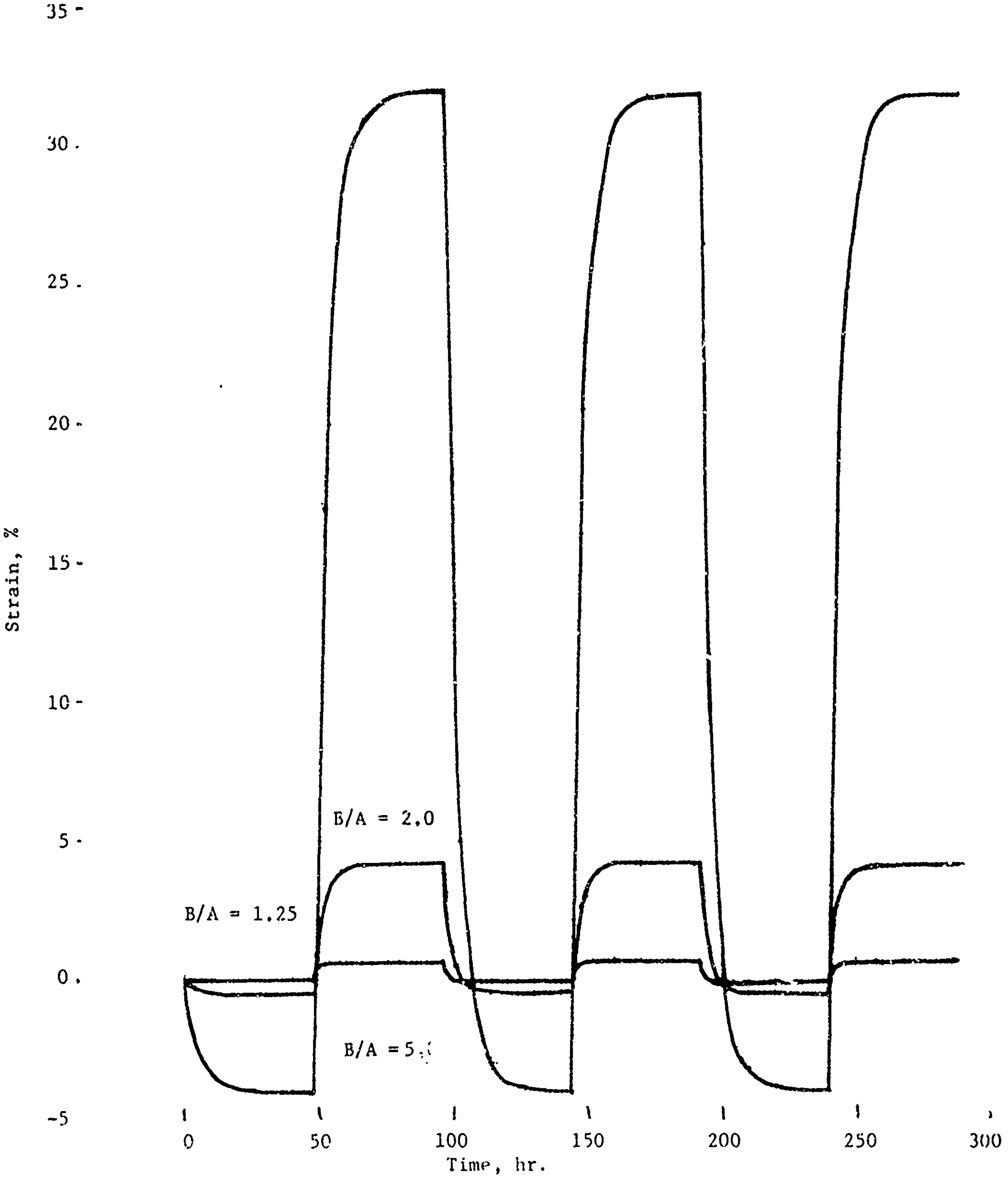

Figure C-1

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 2
(B = 8 in.)

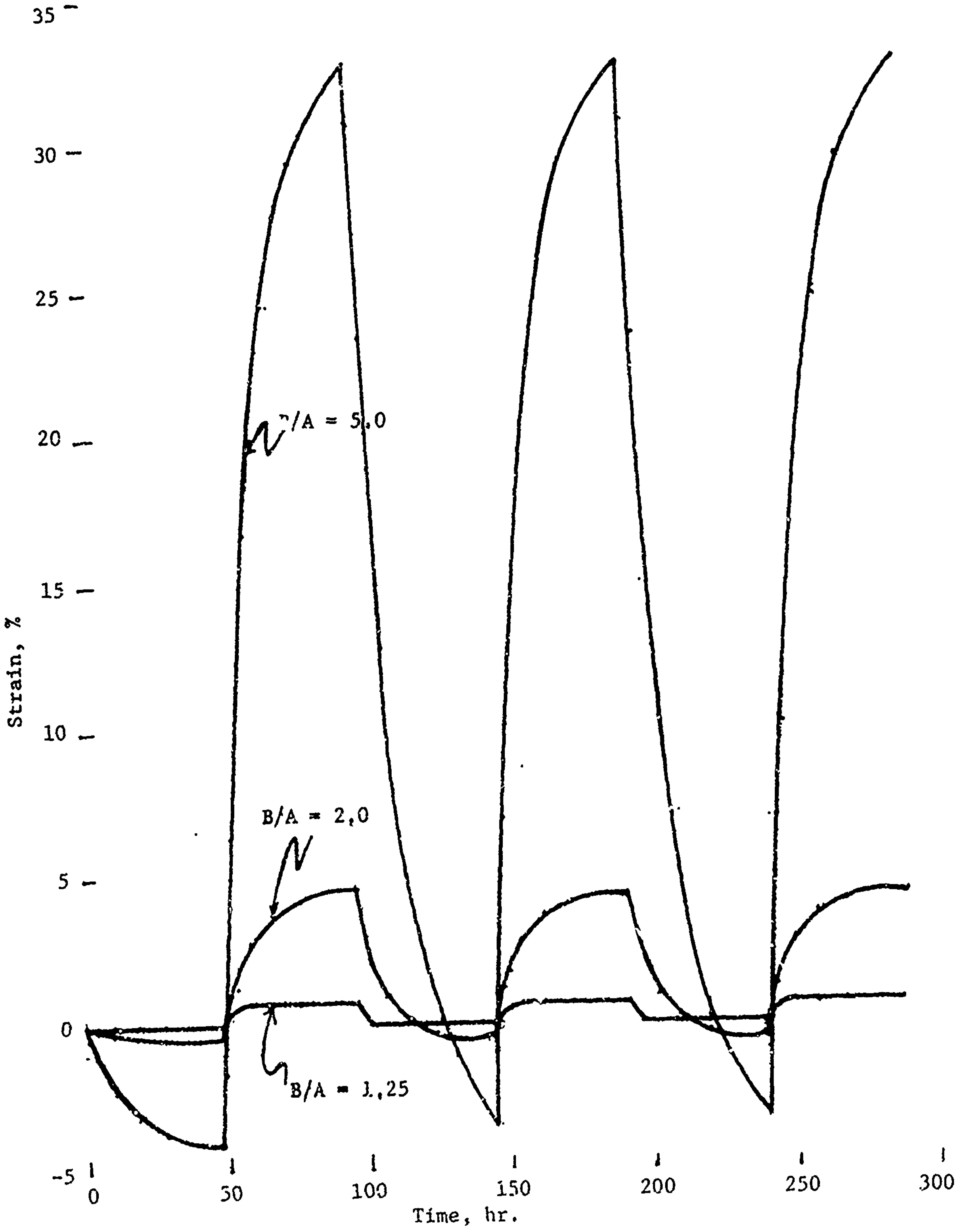

Figure C-2

PARAMETER STUDY OF INNER-BORE HOOP STRESS
CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

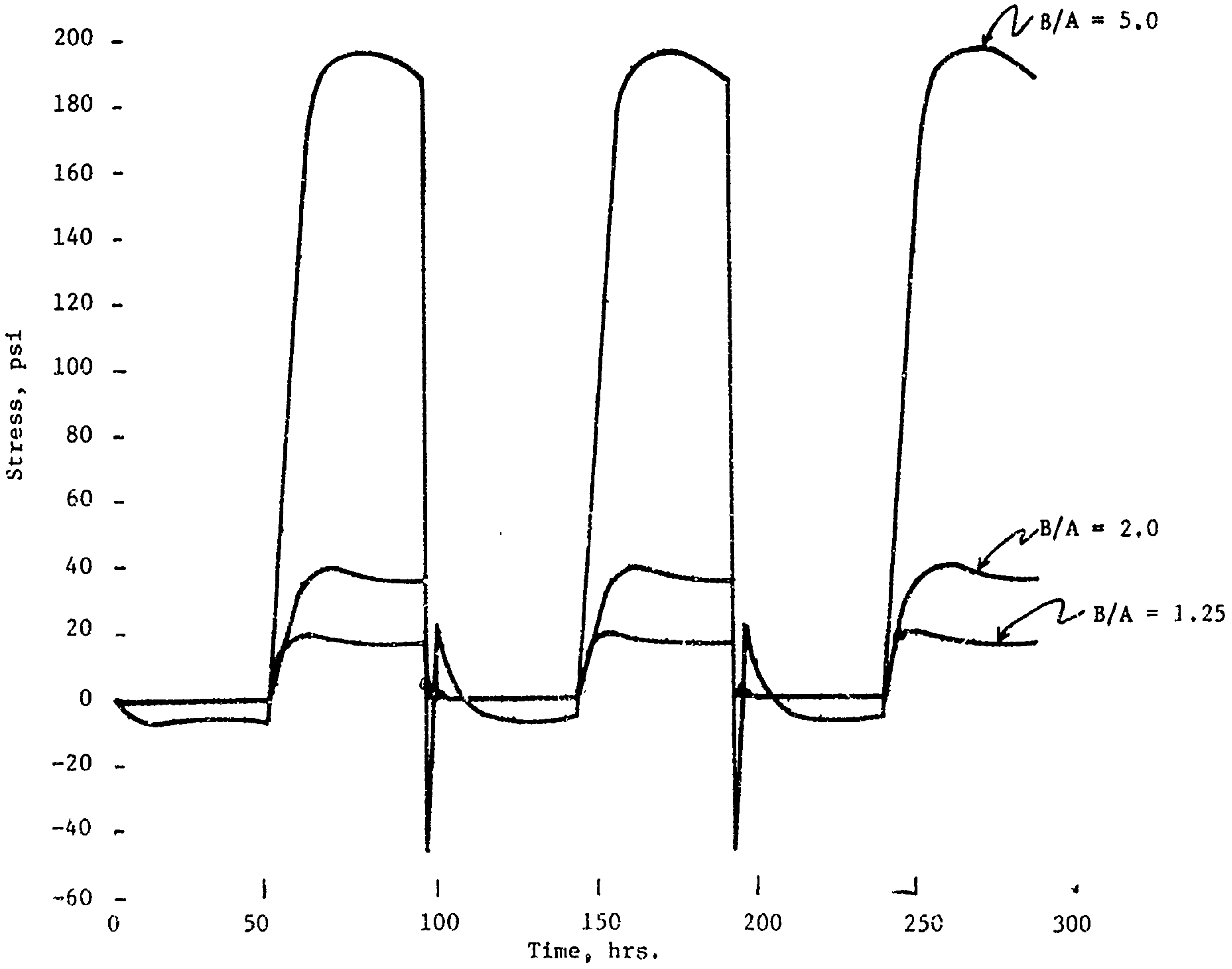

Figure C-3

PARAMETER STUDY OF INNER-BORE HOOP STRESS
CTPB PROPELLANT - HISTORY 2

(B = 8 in.)

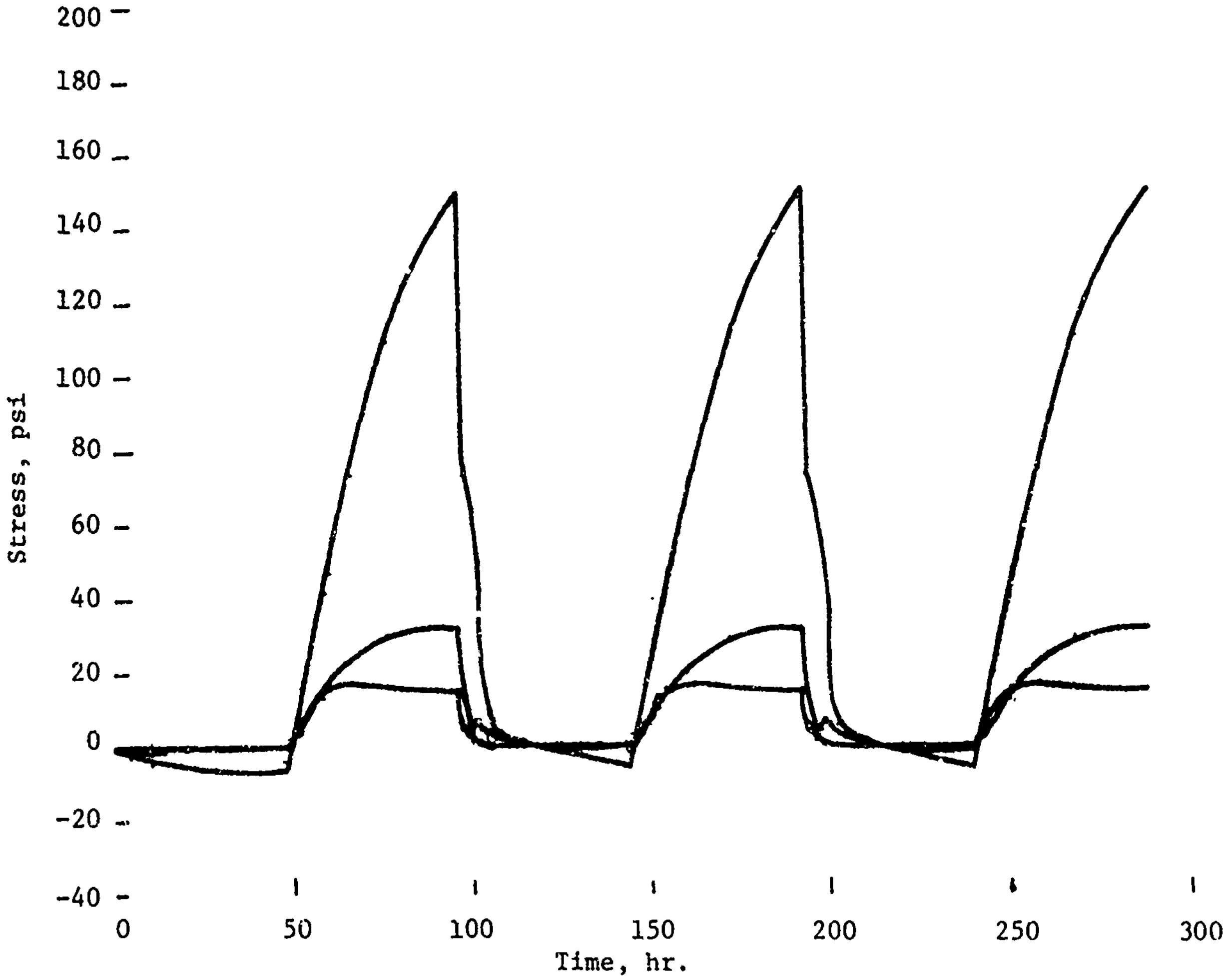

Figure C-4

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

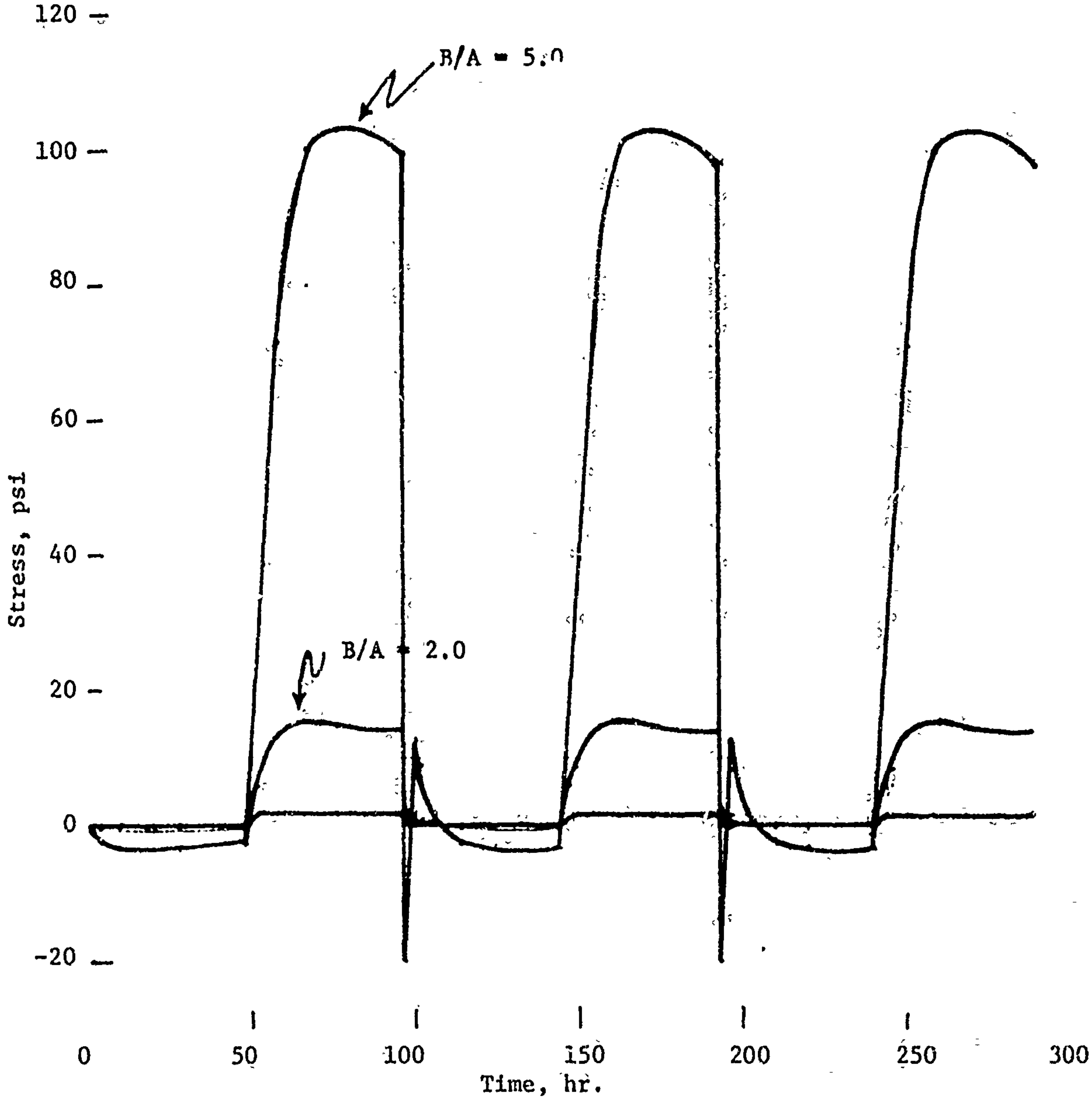

Figure C-5

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 2

(B = 8 in.)

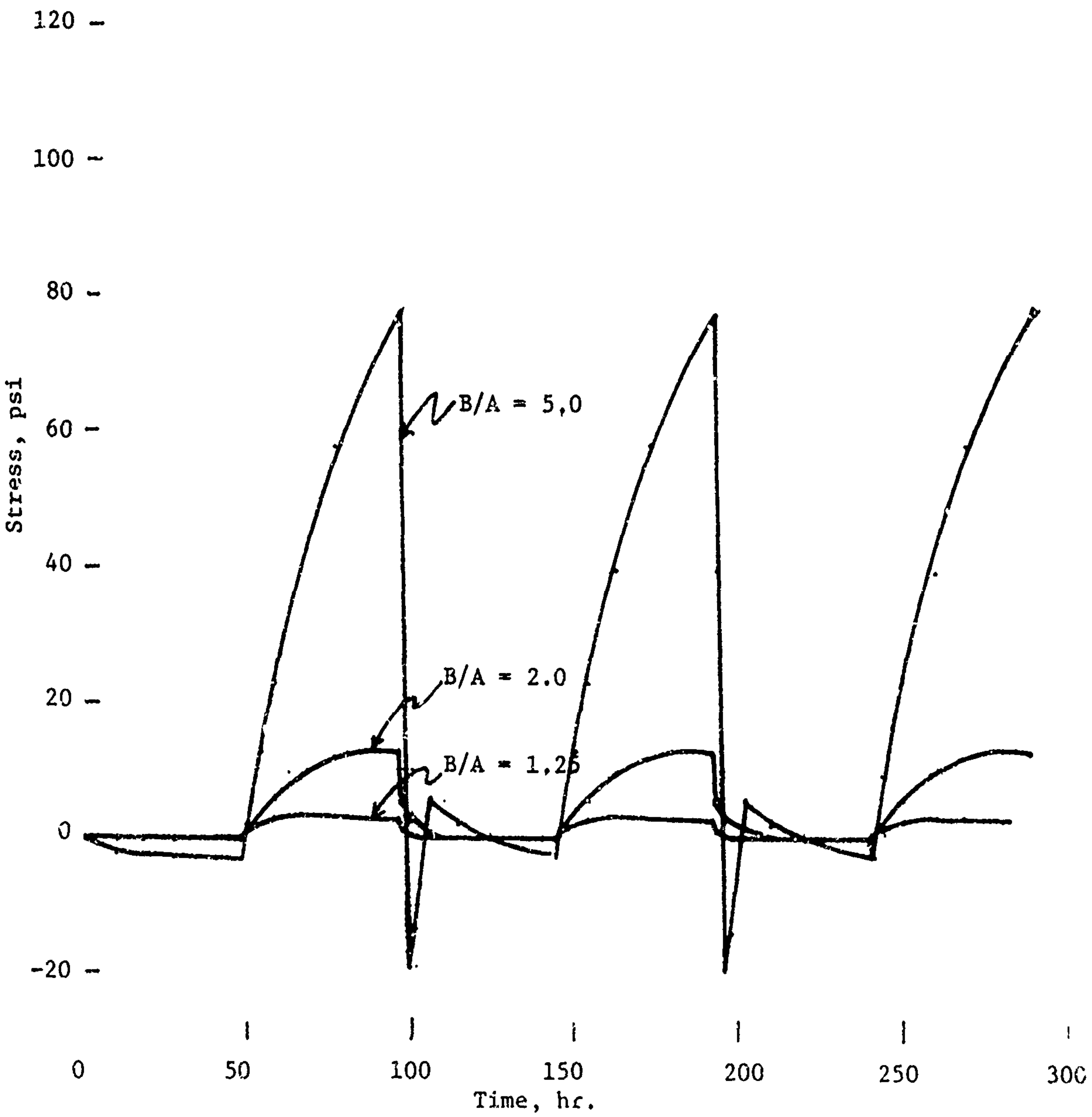

Figure C-6

B. HTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 2
(B = 4 in.)

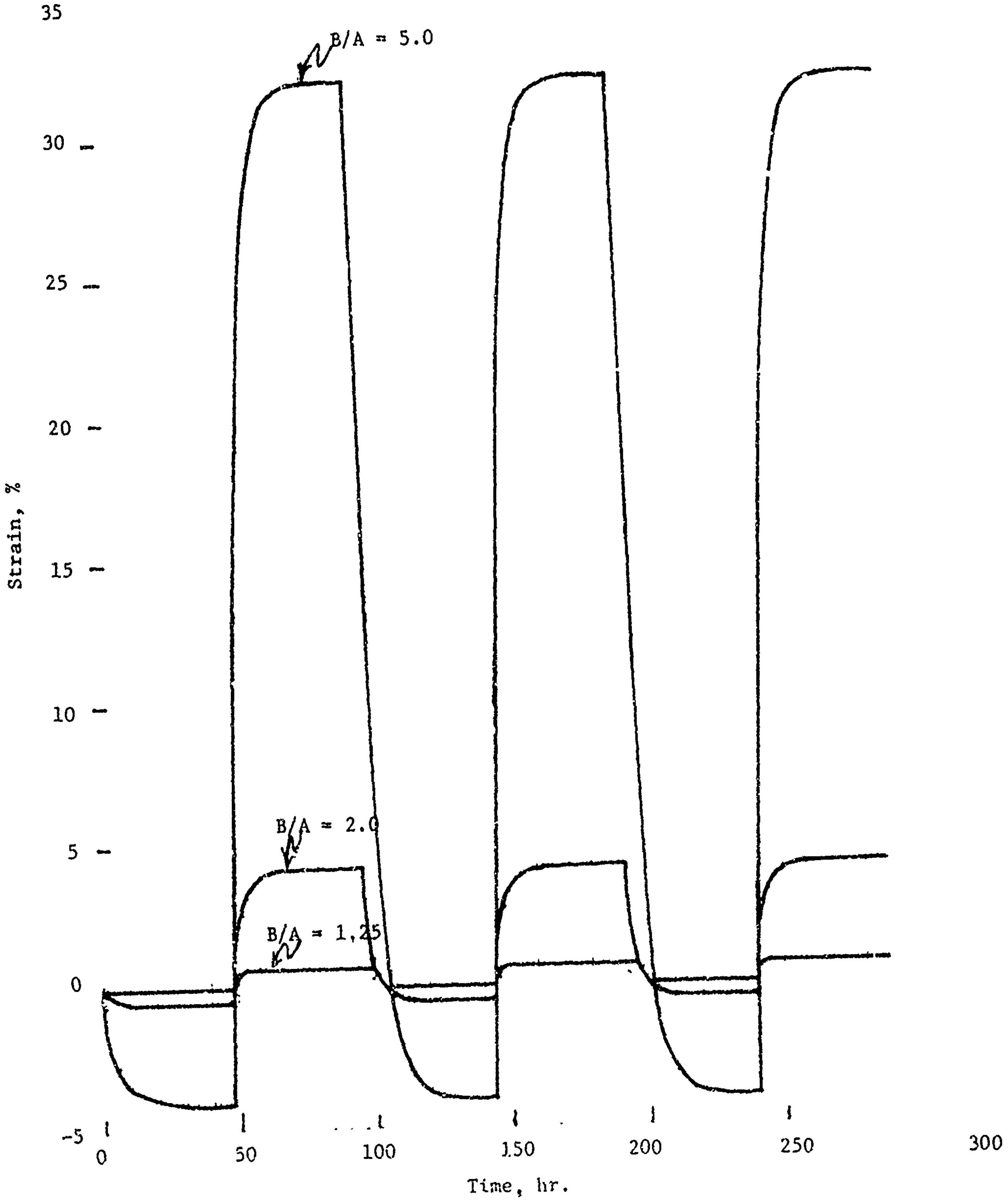

Figure C-7

Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 2

(B = 8 in.)

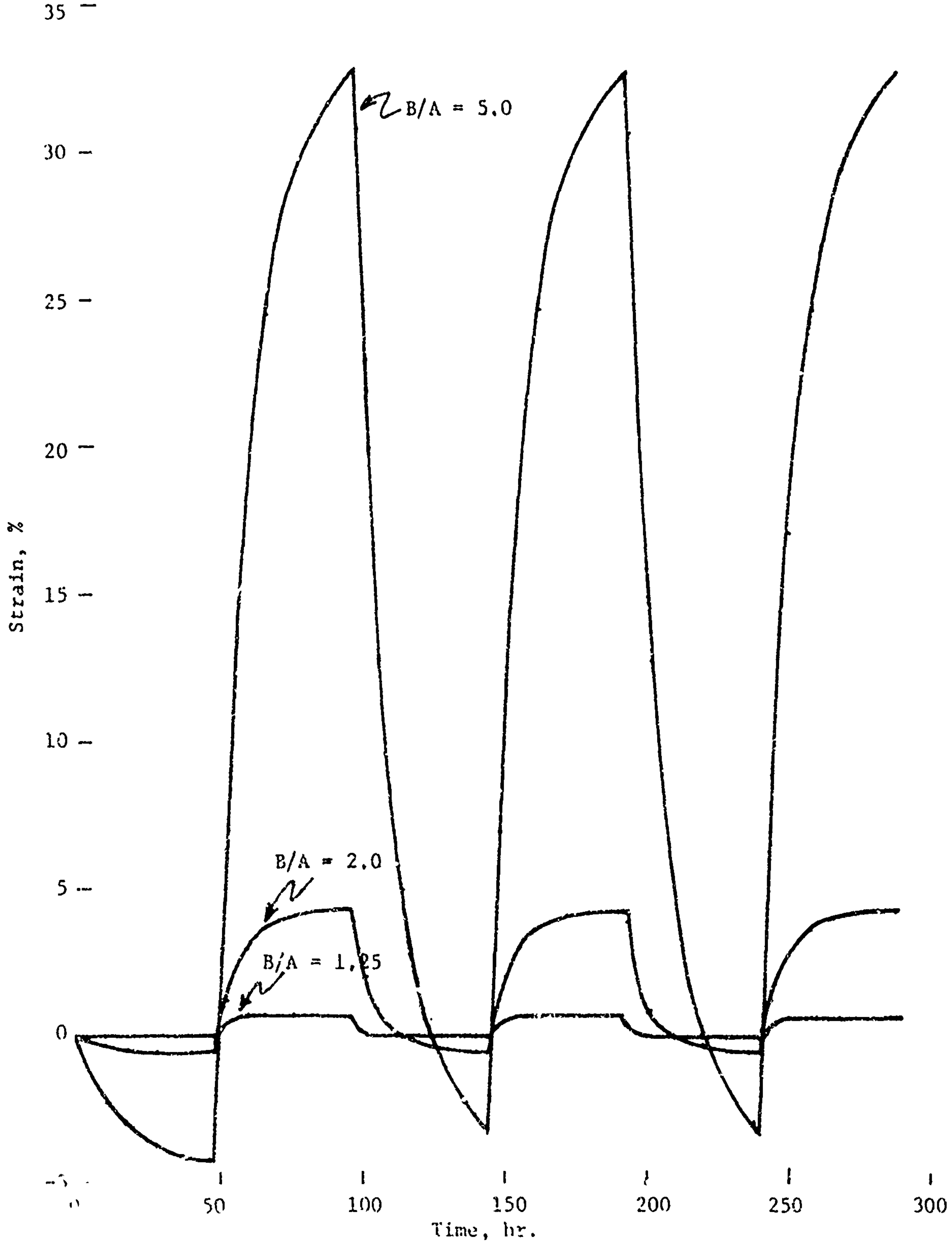

Figure C-8

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 4 in.)

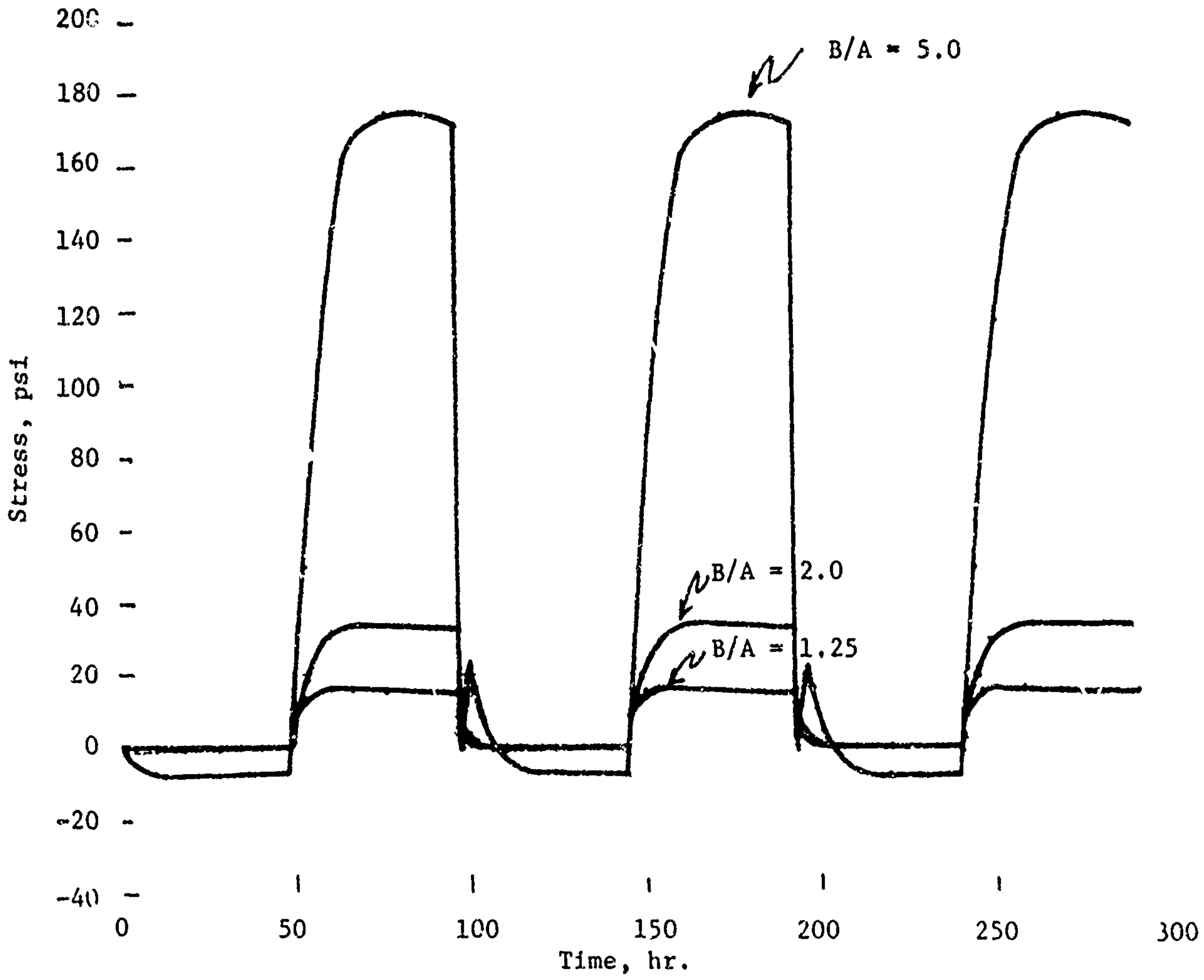

Figure C-9

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 8 in.)

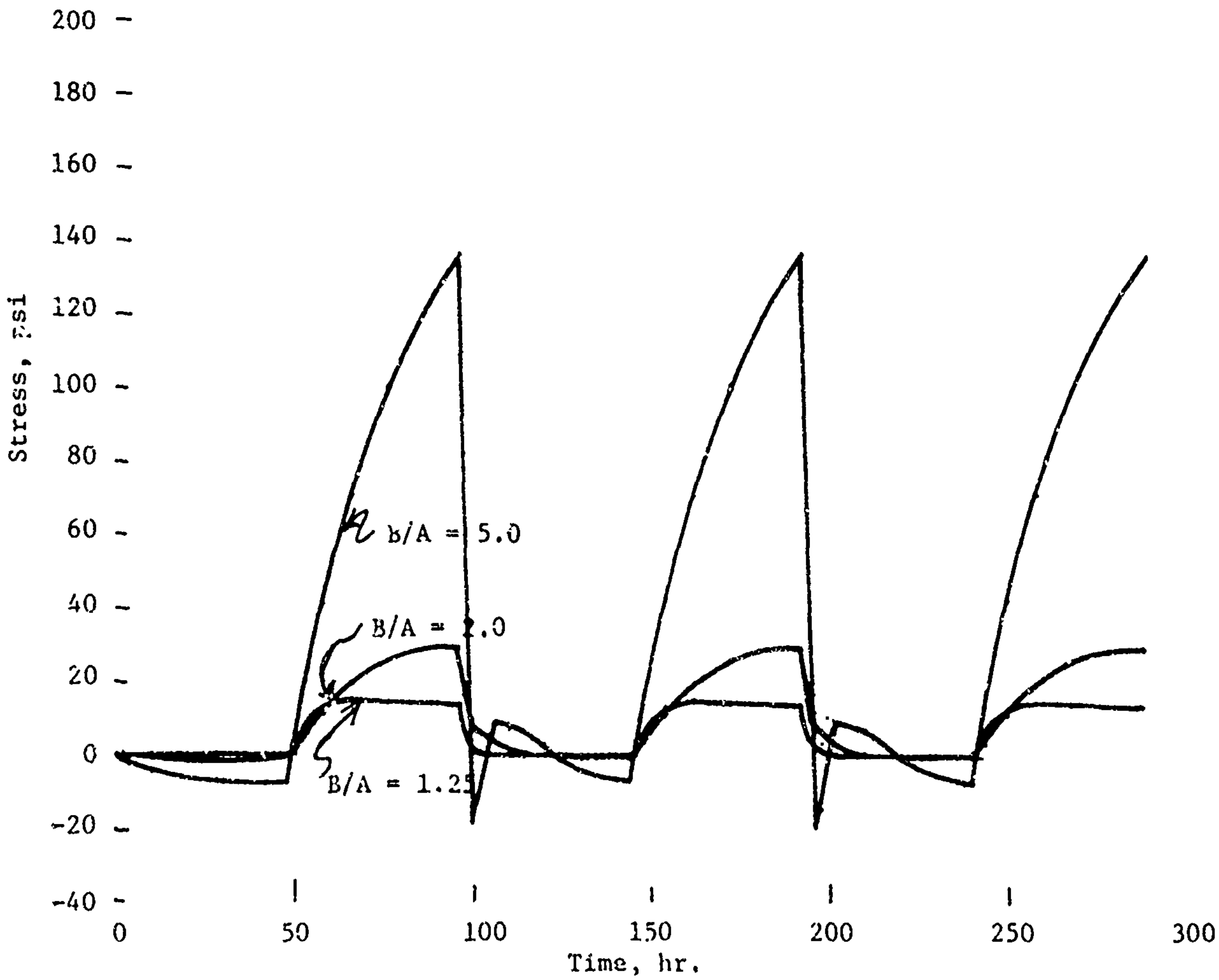

Figure C-10

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 4 in.)

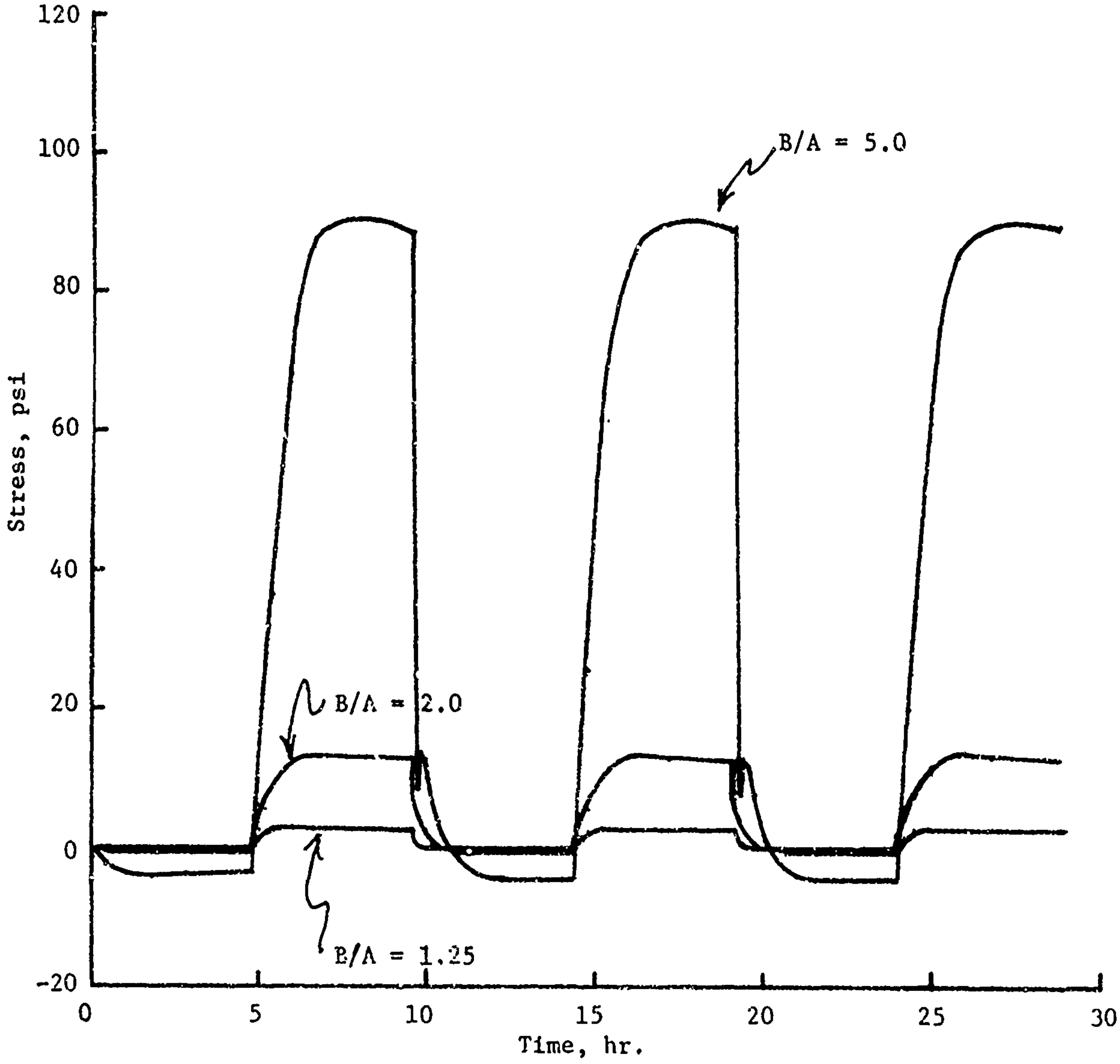

Figure C-11

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 8 in.)

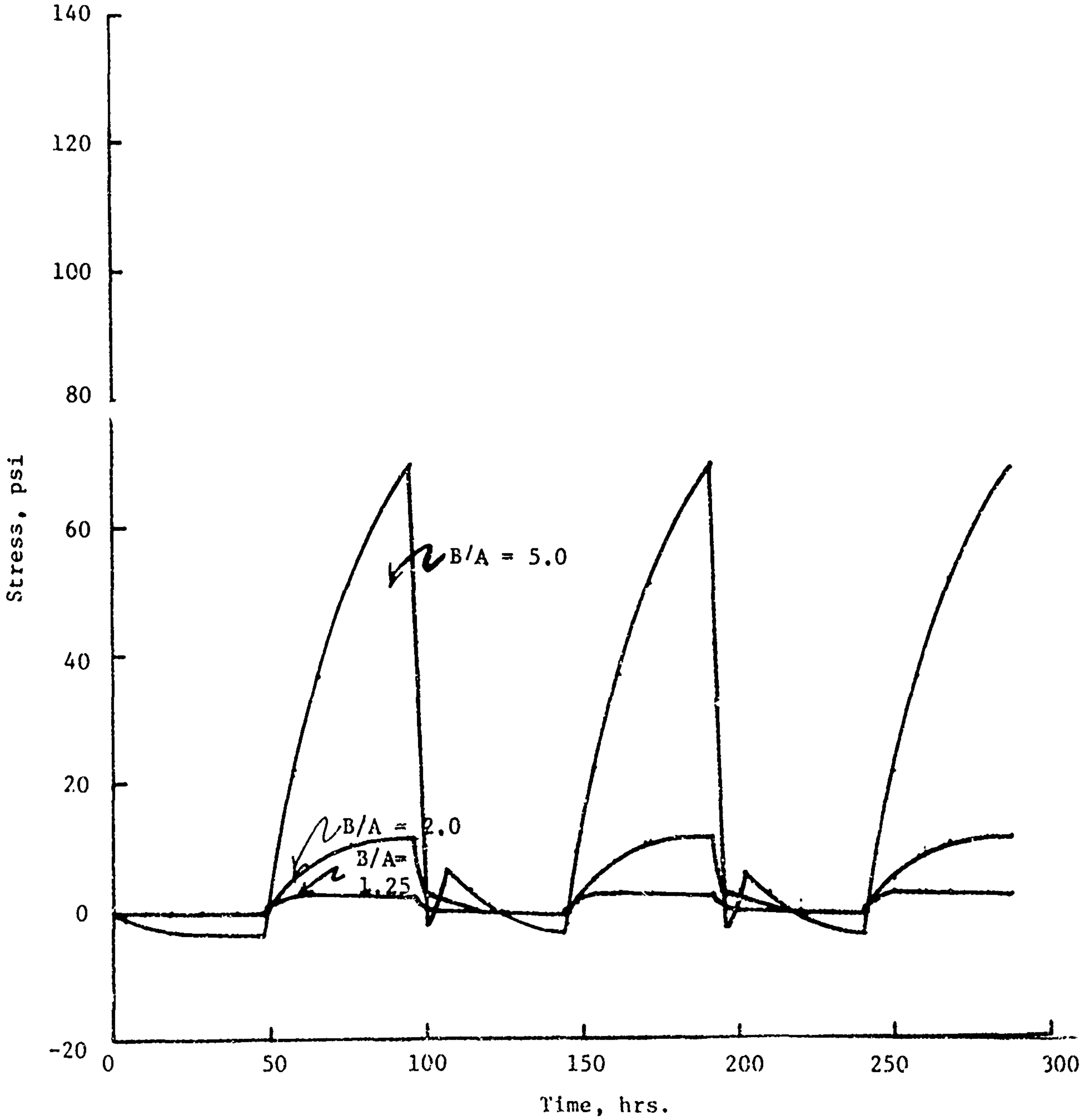

Figure C-12

APPENDIX D

A NEW NORMALIZED RELATION FOR THE RELAXATION MODULUS

APPENDIX D

A NEW NORMALIZED RELATION FOR THE RELAXATION MODULUS

A. NORMALIZATION OF THE SHEAR MODULUS

Conventional viscoelastic stress analyses involve the relaxation modulus in shear, $\mu(t)$. The basic relations for this modulus require difficult to perform experiments, the results of which are not always satisfactory. It was desirable, therefore, to devise a simpler to use expression for this modulus. This was done starting with the relation now used,

$$\mu(t) = \mu_e + (\mu_g - \mu_e) \int_{-\infty}^{\infty} he^{-t/\tau} \, d\ell n\tau \qquad \text{(D-1)}$$

where μ_g is the glassy shear modulus

μ_e is the equilibrium shear modulus

h represents a continuous distribution of relaxation times (normalized)

t is the time of the test

τ is a relaxation time

Since μ_g is difficult to evaluate experimentally an attractive substitute was sought and found.

We solved for $\mu(t)$ at some specific time, like one minute, to obtain $\mu(1)$.

$$\mu(1) = \mu_e + (\mu_g - \mu_e) \int_{-\infty}^{\infty} he^{-1/\tau} \, d\ell n\tau \qquad \text{(D-2)}$$

Letting

$$C = \int_{-\infty}^{\infty} he^{-1/\tau} \, d\ell n\tau \qquad \text{(D-3)}$$

where C is a constant, we solve for $\mu_g - \mu_e$ in Equation (D-2) using Equation (D-3) and inserting the result into Equation (D-1) gives

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \int_{-\infty}^{\infty} \frac{he}{C}^{-1/\tau \, d\ell n\tau} \qquad \text{(D-4)}$$

For practical experimental purposes the constant C can be combined with h to give h', a quantity which is experimentally identified in the same manner as h.

Thus, the new relation becomes

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \int_{-\infty}^{\infty} h' e^{-t/\tau} \, d\ell n\tau \qquad \text{(D-5)}$$

In engineering practice is is unnecessary to evaluate h'. Instead, a graphical plot of $\mu(t)$ versus time is usually sufficient. When normalized results are required a plot of $\dfrac{\mu(t) - \mu_e}{\mu(1) - \mu_e}$ versus the time is equivalent to a plot of $\int_{\infty}^{\infty} h' e^{-t/\tau} \, d\ell n\tau$ versus the time.

Obviously, a broad range of relaxation curves can be obtained for a given distribution of relaxation times, h'.

B. NORMALIZATION OF THE PRONY SERIES

The new relation permits a normalization of the Prony Series as well. This relation in its usual form is given as

$$\mu(t) = \alpha_o + \sum_{m=1}^{m} \alpha_m e^{-\beta_m t} \qquad \text{(D-6)}$$

where the α_m and β_m are constants

α_o is the equilibrium relaxation modulus

We can normalize Equation (D-6) to give a form similar to that of Equation (D-5). First we replace α_o by the equivalent term μ_e then normalize the constants α_m, as shown below, to give

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \sum_{m=1}^{m} \alpha'_m e^{-\beta_m t} \qquad \text{(D-7)}$$

where

$$\alpha'_m = \alpha_m / (\mu(1) - \mu_e) \qquad \text{(D-8)}$$

Equation (D-7) forms the engineering basis of our normalization procedures. This normalization method defines the modulus in terms of two easily determined parameters, μ_e and $\mu(1) - \mu_e$. These same parameters can be used to normalize the stress and strain data from our engineering analyses.

APPENDIX E

INCREMENTAL ANALYSIS PROCEDURE

APPENDIX E

INCREMENTAL ANALYSIS PROCEDURE

In an attempt to minimize accumulated numerical errors in our linear viscoelastic analyses, Dr. Herrmann developed this new approach. The "total stress analysis" was replaced by an "incremental analysis procedure"; i.e., instead of solving for the total stress and strain in the propellant for a given point in time, one could solve instead for the incremental changes in stress and strain. The incremental equations are very similar to those previously reported for the "total analysis" with the following exceptions (the equations referred to by number are reported in Reference (E-1).

Consider first, Equation (13),

$$S_{ijN} = 2\mu_N \, e_{ijN} + L_{ijN} \tag{13}$$

The quantities S_{ij_N} and e_{ij_N} in Equation (13) need to be interpreted as the incremental changes in stress and strain during time step N (i.e., $S_{ij_N} = 2\mu_N \Delta e_{ij_N} + L_{ij_N}$ where $S_{ij_N} = S_{ij_{N-1}} + \Delta S_{ij_N}$, etc.). Equation (18)

$$L_{ijN} = 2\,[\chi_{ijN} - (\mu_N - \alpha_o)\, e_{ijN-1}] \tag{18}$$

is replaced by

$$L_{ij_N} = 2 \sum_{m=1}^{M} \alpha_m \,[e^{-\beta_m \xi_N} - 1]\, C_{ij_{Nm}}$$

Equation (19)

$$X_{ijN} = \sum_{m=1}^{M} \alpha_m C_{ijNm} \qquad (19)$$

is eliminated and Equation (24)

$$C_{ijNm} = e^{-\beta_m \Delta\xi_N} [C_{ijN-1,m} + (e_{ijN-1} - e_{ijN-2}) J_{N-1,m}] \qquad (24)$$

becomes

$$C_{ij_{Nm}} = e^{-\beta_m \Delta\xi_{N-1}} C_{ij_{N-1,m}} + \Delta e_{ij_{N-1}} J_{N-1,m} \qquad (24)$$

Finally, Equations (1), (2) and (3)

$$\tau_{ij} = S_{ij} + \delta_{ij} \sigma, \quad \sigma = \frac{1}{33} \tau_{ii} \qquad (1)$$

$$\varepsilon_{ij} = e_{ij} + \delta_{ij} \frac{\theta}{3}, \; \theta = \varepsilon_{ii} \qquad (2)$$

$$\sigma = K (9 - 3\alpha\Delta T) \qquad (3)$$

where K = Bulk modulus
α = Coefficient of linear thermal expansion
$\Delta T = T(x, t) - T_o$
T_o = Initial stress free reference temperature

are replaced by their corresponding incremental forms (Note: $\Delta T_N = T_N - T_{N-1}$).

REFERENCES

E-1 Herrmann, L. R., and Peterson, F. E., "A Numerical Procedure for Viscoelastic Stress Analysis", Bulletin of the 7th Meeting of the ICRPG Mechanical Behavior Working Group, CPIA Publication No. 177, p. 155 (October 1968).

APPENDIX F

PRONY SERIES CURVE FIT ANALYSIS

APPENDIX F

PRONY SERIES CURVE FIT ANALYSIS

The shear relaxation modulus for most solid propellants has been found to fit the following series function with relatively few terms:

$$\phi(t) = A_o + \sum_{i=1}^{n} A_i e^{-\beta_i t} \qquad \text{(F-1)}$$

Equation (F-1)is a "Prony" series with two unknown coefficients A_i and β_i. The method of collocation is used to find these coefficients:

$$\text{Let } \beta_i = \frac{1}{2t_i} \qquad \text{(F-2)}$$

Substituion of Equation (F-2) into Equation (F-1) gives

$$\phi = A_o + \sum_{i=1}^{n} A_i e^{-\frac{t}{2t_i}} \qquad \text{(F-3)}$$

Now, choose n points for the evaluation of ϕ

$$\phi_j = A_o + \sum_{i=1}^{n} A_i e^{-\frac{t_j}{2t_i}} \qquad (J = 1,n) \qquad \text{(F-4)}$$

Equations (F-4) are sufficient to solve for the A_i. In matrix notation;

$$[\overline{E}]\{A_i\} = \{\phi_j - A_o\} \qquad \text{(F-5)}$$

where,

$$E_{ji} = e^{-\frac{t_j}{2t_i}}$$

Equation (F-5) is readily solved for A_i;

$$\{A_i\} = [E]^{-1}\{\phi_j - A_o\} \qquad \text{(F-6)}$$

Equation (F-6) has been programmed for computer solution from a times burning terminal in the BASIC language. A listing of the program is given on Page F-3. The order of data input is given below:

1. n
2. A_o
3. t_j ($j = 1, n$)
4. ϕ_j ($j = 1, n$)

Data statements 600 to 9000 may be used for data. Sample data statements are shown below:

```
600 Data 7,100

610 Data 1E-4, 1E-3, 1E-2, 1E-1, 1, 10, 100

620 Data 3000, 1800, 1000, 620, 410, 320, 280
```

A sample run with this data is shown on Page F-4.

PROGRAM LISTING

```
100 REM "PRONY SERIES CURVE FIT"
110 DIM T(20,1),E(20,1),A(20,20),C(20,20)
120 DIM F(1,20),G(1,20)
130 READ N,E1
140 MAT READ T(N,1)
150 MAT F = ZER(1,N)
160 MAT G = ZER(1,N)
170 MAT F = TRN(T)
180 PRINT "INPUT TIMES AND MODULI"
190 MAT PRINT F;
200 MAT A = ZER(N,N)
210 FOR J=2 TO N
220 LET E = -T(J,1)/T(1,1)
230 LET A(J,J) = 0.60653
240 IF E < -20.0 THEN 280
250 LET A(1,J) = EXP(0.5/E)
260 LET A(J,1) = EXP(0.5*E)
270 GO TO 300
280 LET A(1,J) = 1.0
290 LET A(J,1) = 0.0
300 NEXT J
310 LET A(1,1) = 0.60653
320 LET K1 = N - 1
330 FOR I=2 TO K1
340 LET L = I
350 LET K2 = K1 - I + 2
360 FOR J=2 TO K2
370 LET L = L + 1
380 LET A(J,L) = A(1,I)
390 LET A(L,J) = A(I,1)
400 NEXT J
410 NEXT I
420 MAT C = ZER(N,N)
430 MAT C = INV(A)
440 MAT READ E(N,1)
450 MAT G = TRN(E)
460 MAT PRINT G;
470 FOR J=1 TO N
480 LET E(J,1) = E(J,1) - E1
490 NEXT J
500 MAT T = C*F
510 PRINT "SOLUTION"
520 MAT F = TRN(T)
530 MAT PRINT F;
540 GO TO 440
550 END
[illegible]
```

SAMPLE RUN

```
PR0N        9:57   SF FRI 02/13/70

INPUT TIMES AND M0DULI
 .0001    .001  .01   .1    1  10     100


 3000  1800  1000  620   410   320   280


S0LUTI0N
 1370.27  1031.23  464.66  247.646  184.314 -106.615  297.955



0UT 0F DATA  IN 440
```

APPENDIX G

INCLUSION OF NON-ZERO THICKNESS STRESSES IN PLANE STRESS ANALYSES

APPENDIX G

INCLUSION OF NON-ZERO THICKNESS STRESSES IN PLANE STRESS ANALYSES

The generalized plane stress option was modified so that the stress throughout the thickness of the body, may be specified as a constant rather than zero. The values of the thickness stress constitutes one of the input parameters. The inclusion of a non-zero thickness stress required the modification of the governing variational equation.

It was determined that the appropriate form of the variational function, for a nearly incompressible material(G-1) is (for $T = T_N$):

$$
\begin{aligned}
F_N = \iint \Big\{ & \frac{2\mu_N}{3} \left[(\Delta\varepsilon_{x_N})^2 + (\Delta\varepsilon_{y_N})^2 - (\Delta\varepsilon_{x_N})(\Delta\varepsilon_{y_N}) \right] + \frac{1}{2} \mu_N \, (\Delta\gamma_{x4_N})^2 \\
& + (\mu_N \, \Delta\bar{H}_N - \Delta\beta_N/3)(\Delta\varepsilon_{x_N} + \Delta\varepsilon_{y_N}) + L_{xx_N} \, \Delta\varepsilon_{x_N} + L_{yy_N} \, \Delta\varepsilon_{y_N} \\
& + L_{xy_N} \, \Delta\gamma_{xy_N} - \frac{(\mu_N)^2}{2\bar{K}_N} (\Delta\bar{H}_N)^2 + \Delta\bar{\bar{H}}_N \left(\frac{\Delta\beta_N}{2} - 3\mu_N \, \alpha \, \Delta T_N \right) \\
& - \Delta F_{x_N} \, \Delta u_{x_N} - \Delta F_{y_N} \, \Delta u_{y_N} \Big\} \, dxdy - \int \Delta u_N \cdot \Delta(\text{applied boundary load}) \, ds
\end{aligned}
$$

where

$$\Delta\sigma_{xx_N} = \frac{2\mu_N}{3} \left(2\Delta\varepsilon_{x_N} - \Delta\varepsilon_{y_N}\right) + \mu_N \; \Delta\bar{H}_N + L_{xx_N} - \frac{\Delta\beta_N}{3}$$

$$\Delta\varepsilon_{z_N} = \frac{1}{3} \left(\Delta\varepsilon_{x_N} + \Delta\varepsilon_{y_N}\right) - \frac{\Delta\Delta\bar{H}_N}{2} + \frac{2}{3\mu_N} \left(\Delta\sigma_N - L_{zz_N}\right) + \frac{1}{3} \; \alpha \; \Delta T_N$$

$$\Delta\beta_N = 2\mu_N \; \alpha \; \Delta T_N + \left(\Delta\sigma_N - L_{zz_N}\right)$$

$$\bar{K}_N = \frac{8\mu_N(1 - \frac{\mu_N}{6K})}{3(1 + \frac{4\mu_N}{3K}}$$

The symbol $\Delta\sigma_N$ denotes the incremental change in the specified thickness stress, L_{zz_N} denotes the history term associated with ε_z, the other symbols have their usual meanings.

REFERENCES

G-1 Herrmann, L. R., "Elasticity Equations for Incompressible and Nearly Incompressible Materials by a Variational Theorem", AIAA J., Vol. 3, No. 10, pp. 1896-1901, October 1965.

APPENDIX H

A COMPUTER PROGRAM FOR VISCOELASTIC SOLIDS OF REVOLUTION SUBJECTED TO TIME - VARYING THERMAL AND MECHANICAL LOAD ENVIRONMENTS

VERSION 2.1

Appendix H

A COMPUTER PROGRAM FOR VISCOELASTIC SOLIDS OF REVOLUTION SUBJECTED TO TIME-VARYING THERMAL AND MECHANICAL LOAD ENVIRONMENTS

- VERSION 2.1 -

I. INTRODUCTION

The purpose of the computer code described in this section is to perform viscoelastic stress analyses. The analyses is applicable to arbitrary revolved solids and plane structures subjected to loads of mechanical or thermal origin. The program is segmented into two (2) phases: (1) Transient Heat Transfer Analysis; and (2) Viscoelastic Stress Analysis. The purpose of the heat transfer phase is to generate temperature distributions in the body as a function of time which are used subsequently in performing the stress analysis. The two phases of the program can be used in sequence within a given job or each phase can be used separately.

The method of solution employs the finite element procedure for solving the spatial problems (heat conduction and stress analysis) and time marching techniques to evaluate temperatures/stresses at successive points in time. The transient heat transfer problem is solved using the procedure developed by Wilson and Nickell*. Knowing thermal and mechanical loads as a function of time and having available the viscoelastic properties of the material(s), a set of equivalent elastic parameters is defined for a particular point in time; the equivalent elastic problem is posed using the procedure given by Herrmann and Peterson**. An elastic stress analysis must be performed at each point in time for which the viscoelastic response of the body is required. The stress analysis problem is solved using a finite element method given by Herrmann.***

II. APPLICATIONS

The principal application of the program is to stress analysis of solid propellant grains maintained in time varying temperature environments. A typical application might be the grain stress analysis of a motor system subjected to thermal cycling. Mechanical loads may also be applied to the body either isothermally or with simultaneous time varying temperature changes. The kinds of mechanical loading considered in the analysis include surface pressures, body forces (spin and/or axial accelerations), concentrated nodal forces, and specified nodal displacements; all mechanical loads can vary with time in accordance with user-supplied table.

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* Wilson, E. L., and Nickell, R. E., "Application of the Finite Element Method to Heat Conduction Analysis", Nuclear Engineering and Design 4 (1966), p. 276-286. North Holland Publishing Company, Amsterdam.

** Herrmann, L. R., and Peterson, F. E., "A Numerical Procedure for Viscoelastic Stress Analysis", CPIA - 7th Meeting, Working Group on Mechanical Behavior (ICRPG), November 1968.

*** Herrmann, L. R., "Elasticity Equations for Incompressible Materials by A Variational Theorem", Journal of the AIAA, p. 1896-1900, October 1965.

A. PHYSICAL PROPERTIES

The program can be used to perform simple elastic solutions, time-dependent elastic or thermoelastic analyses, or thermoviscoelastic analyses. A viscoelastic analysis requires a complete material property characterization including the master relaxation curve and shift function for each time dependent material, bulk modulus, expansion coefficient, density, etc. Table H-1 illustrates the thermal and mechanical properties data required to perform the thermoviscoelastic stress analysis of a bipropellant grain with a separate viscoelastic liner and an elastic case.

Shift function data is accepted by the program in the form of a table of log a_T versus T, (°F); shift factors at temperatures other than those supplied in the input table are determined using linear interpolation between given points.

Relaxation data must be input as the coefficients (A_i, β_i) in a Prony Series fit to the experimental data. The relaxation behavior in shear must be expressed as an exponential series.

$$\phi(t) = A_o + \sum_{i=1}^{n} A_i e^{-\beta_i t} \qquad \text{(H-1)}$$

where A_o is the shear equilibrium modulus and (A_i, β_i) are found from curve fitting* calculations based on experimental data. Each viscoelastic material must have its relaxation behavior expressed in a separate series expansion. Elastic materials have no terms in the series other than A_o. If there are no terms in the series expansion (M = 0) the program will not read a shift function table.

The special problem of bulk rapid pressurization of a propellant grain where a pressure shift function a_p is required is handled in a different manner. This type of problem can only be run isothermally with no superimposed thermal loads. In this case the shift function input values are interpreted as the product $a_p a_T$ as a function pressure where a_T is a constant for the temperature under consideration.

---

* One recommended curve fitting procedure is a "collocation method" originated by Schapery and summarized in the ICRPG Solid Propellant Mechanical Behavior Manual starting at Section 2.2-4 (June 1963). The method involves assuming values for the constants β_i and solving a set of linear simultaneous equations for the constants A_i. A time-share routine called "PRONY" has been programmed to perform the calculations at Aerojet.

PHYSICAL PROPERTY INFORMATION REQUIRED FOR A THERMOVISCOELASTIC STRESS ANALYSIS

PROBLEM IDENTIFICATION:

Propellant No. 1 -
Propellant No. 2 -
Liner -
Case -

Physical Property	Propellant No. 1	Propellant No. 2	Liner	Case
Coefficient of Linear Thermal Expansion ($°F^{-1}$)				
Density (lb-in.$^{-3}$)				
Specific Heat Capacity (btu-lb^{-1} $°F^{-1}$)				
Thermal Conductivity, (Btu-in.$^{-1}$ hr^{-1} $°F^{-1}$)				
Bulk Modulus (lb-in.$^{-2}$)				
Shear Relaxation Modulus (lb.-in.$^{-2}$ vs.hr)	(Table or curve as a function of time)			
Shift Factor (a_T vs. °F)	(Table or curve as a function of temperature)			
Elastic Shear Modulus (lb-in.$^{-2}$)				

Table H-1

III. DATA INPUT DESCRIPTION

This section supplies information necessary for the preparation of data input cards. The input sequence is separated into four major groups:

1. Grid Definition
2. Solution Time and Temperature Information
3. Transient Heat Transfer Solution Data
4. Stress Analysis Information

A. NOMENCLATURE

The abbreviation "cc" used below stands for "card columns". The variable names assigned to the various parameters used by the program are given below in upper case letters; for example, "NNP" stands for the total number of nodes in the finite element mesh. All variables starting with any of the letters I, J, K, L, M, N are to be input to the program as integers (i.e., without a decimal point). All integers are to be packed to the right of the field specified by the "cc" numbers. Any variable whose first letter is not an I, J, ..., N is a real number requiring a decimal; "R(N)", for example, is the radius of the "N-th" nodal point (entered in cc 6-15). If R(N) = 13.45, then the number "13.45" can be placed anywhere in the field: cc 6-15. Real numbers can also be input in "E" format; 13.45 could be entered as 1.345E1, .1345E2, 1345.E-2, etc., providing the set of characters is packed to the right of the cc field.

Whenever applicable, units of the variables are stated in symbolic notations. The symbols used below are defined as follows:

(F) = Force units
(L) = Length units
(T) = Time units
(°F) = Temperature units
(R) = Radian
(Btu) = Thermal Heat Flow Units

Thus, the quantity (F) $(L)^{-2}$ written after the shear modulus means "psi" if pounds and inches are the units chosen by the user. There are no units, conversion factors, etc., built into the program; thus, once a set of units is chosen it must be used consistently throughout the analysis.

B. SEQUENCE OF OPERATIONS

The flow of program execution is controlled by three (3) user-supplied variables:

IPF = Plot Control Flag
NTEM = Temperature Information Flag
JØB = Job Control Flag

Table H-2 contains values which can be assigned to these variables showing what operation results from a particular specification. Certain combinations are not possible. For example, if $IPF < 0$, the program reads mesh data, prepares a plot and returns control to that portion of the program which looks for the next job; thus, the variables "NTEM" and "JOB" cannot be specified. "JOB" has no meaning unless $IPF \geq 0$ and $NTEM = 0$.

The option NTEM = 1 is impractical for real problems because of the amount of card data involved; this option is useful when solving "check cases" with temperature distributions generated from an analytical expression or formula. The NTEM = 2 option saves re-runing the temperature problem if only the mechanical properties or mechanical loads change. NTEM = 3 is used for the isothermal problem in which the loading is mechanical in origin. Viscoelastic materials exhibit temperature dependence, so material temperatures must be defined even though there are no driving thermal strains in the body to be analyzed.

The variable "JØB" (which is only defined if NTEM = 0) controls what the program does with the results of the heat transfer solution and where the program goes after the heat transfer calculations. If JØB = 0, element temperatures are saved temporarily for use in the stress analysis to follow. JØB = 1 results in the same operation as JØB = 0, but in addition the element temperatures are saved for use in a later analysis (or series of analyses). If JØB = 2, the temperatures are saved (and printed at user specified time intervals) on a tape for use at some other time; at this point the program is through with this job and looks for another. JØB = 3 allows the user to run the program solely as a heat transfer analysis.

EXECUTION CONTROL VARIABLES

FLAG	VALUE	OPERATION PERFORMED
IPF	< 0	Read grid data only, plot grid and stop.
	$= 0$	Read all data, execute the job without a plot.
	> 0	Read all data, execute the job with a plot.
NTEM	$= 0$	Calculate the element temperatures using the heat transfer analysis.
	$= 1$	Read the element temperatures from card input.
	$= 2$	Read the element temperatures from a tape created during a previous run.
	$= 3$	Read the element temperatures from card input and use the same distribution for all solution time points (isothermal response).
JOB	$= 0$	Run the heat transfer problem and use the results to perform the stress analysis.
	$= 1$	Run the heat transfer problem, save the element temperatures on a permanent file (which can be used as input to subsequent jobs) and use the results to perform the stress analysis.
	$= 2$	Run the heat transfer problem, save the element temperatures on a permanent file, and stop.
	$= 3$	Run the heat transfer problem and stop.

Table H-2

C. DATA CARD INPUT

1. Grid Definition

a. Start Cards

(1) First Card (A3)

cc	
1-3	Enter the characters "TVA"

(2) Title Card (12A6)

cc		
1-72	HED	Title information for job and plot (center about cc 36 for plot)

b. Control Card (One card, 3I5)

cc				
1-5	NNP	Number of nodal points		≤ 273
6-10	NEL	Number of quadrilateral elements		≤ 240
11-15	IPF	Plot flag	< 0, plot only = 0, run only > 0, plot and run	

c. Node Coordinate Cards (I5, 2F10.3, I5, 2F10.3, I5)

cc			
1-5	N	Node Number	
6-15	R(N)	Radial Coordinate	(L)
16-25	Z(N)	Axial Coordinate	(L)
26-30	NE	Ending Node Point Number	
31-40	R(NE)	Radial Coordinate	(L)
41-50	Z(NE)	Axial Coordinate	(L)
51-55	NI	Node Number Increment	

If the ending node number NE is not zero (or blank), then nodes will be generated in equal distance increments along a line between node N and node NE. The first generated node is assigned the number N + NI; the second generated node number is N + 2NI, etc. Note that the node number difference (NE-N) must be positive and divisible by NI.

If NI is omitted it will be assigned a value of "1" automatically.

d. Element Numbering Cards (8I5)

cc		
1-5	N	Element Number
6-10	IX (N, 1)	Node numbers describing the corner points of the quadrilateral
11-15	IX (N,2)	
16-20	IX (N,3)	
21-25	IX (N,4)	
26-30	MN(N)	Material Number
31-35	NLG	Number of elements to be generated
36-40	NNI	Node Number Increment

If the number of elements to be generated (NLG) is not zero, then NLG elements will be generated. The first generated element is assigned the number N + 1; the second generated element is numbered N + 2, etc. The node numbers of the second generated element are found by adding NNI to the node numbers of the first generated element, etc. If NNI is omitted it will be assigned a value of "1".

The material number assigned to generated elements is the same as that for element N. Material numbers are not necessary if the program is to plot the grid only (IPF< 0).

If IPF < 0; data input ends here.

2. Solution Time and Temperature Information

a. Control Card (One card 2I5, F10.0, 2I5, F10.0).

cc		
1-5	NDT	Total number of time increments to be used in the solution for element temperatures and/or stresses.
6-10	NTR	Number of regions on the time axis having the same value of time increment ratio. ≤ 50
11-20	TMF	Value of time at the end of the NDT$^{\underline{th}}$ increment. (T)
21-25	NBCF	Number of functions describing time dependent boundary conditions. ≤ 10
26-30	NTEM	Temperature information flag: = 0 Element temperatures are to be calculated using the heat transfer analysis. = 1 Element temperatures are to be read from cards. = 2 Element temperatures are to be read from a tape created on a previous run. = 3 Element temperatures are to be read from cards, and this distribution is to be used for all solution time points.
31-40	TZ	Temperature of every element at time zero (°F) (stress free temperature).

b. Solution Time Points

At least one card in this section

(1) First Card (3 (I5,2FL0.0)

cc		
1-5	NTI(1)	Number of increments for which the time increment ratio C(1) remains constant in region 1.
6-15	DT (1)	Value of time after NTI(1) time increments in region 1.
16-25	C (1)	Time increment ratio in region 1.
26-30	NTI (2)	Same as cc 1-5 for region 2.
31-40	DT (2)	Value of time after NTI (1) HNTI (2) time increments (T)
41-50	C (2)	Same as cc 16-25 for region 2
51-55	NTI (3)	
56-65	DT (3)	Region 3
66-75	C (3)	

(2) Second Card (3 (I5, 2 F10.0)) (If required)

cc		
1-5	NTI (4)	
6-15	DT (4)	Region 4
16-25	C (4)	
	...	Etc.

Use as many cards in this section as are required to enter NTR groups of (NTI (I), DT (I), C (I)). Note that

$\sum_{i=1}^{NTR} NTI_i = NDT$, otherwise an error message will be issued by the program.

The solution time points in region r are found using:

$$t_{i+1} = t_i + C_r^* (t_i - t_{i-1})$$

This method of solution time point input has the effect of automatically producing small time steps at the beginning of the region while continuously widening the time steps toward the end of the region. As an example, if 50 solution time points are specified over a 24 hour period with C = 1.1 the first solution time point will occur at .0206 hrs (1.24 minutes) and the last solution step $(t_{50} - t_{49})$ will be 2.19 hours (132 minutes).

It is important to choose C such that the smallest time step will not be so small as to produce a reduced time falling off the mechanical properties table or the final time step too large. The following equations may be used to find these values:

$$\frac{\Delta T_i}{T_f} = \frac{C-1}{C^N-1} \qquad \frac{\Delta T_N}{T_f} = \frac{C^{N-1}\,(C-1)}{C^N - 1} \qquad \text{(H-2)}$$

where

C = time increment ratio
ΔT_i = Initial solution time step
Δ N = total number of solution time points
ΔT_N = final time step
T_f = final value of time ($T_o = 0$)

These two equations are plotted for various values of C in Figures H-1 and H-2 respectively.

INITIAL SOLUTION TIME INCREMENT

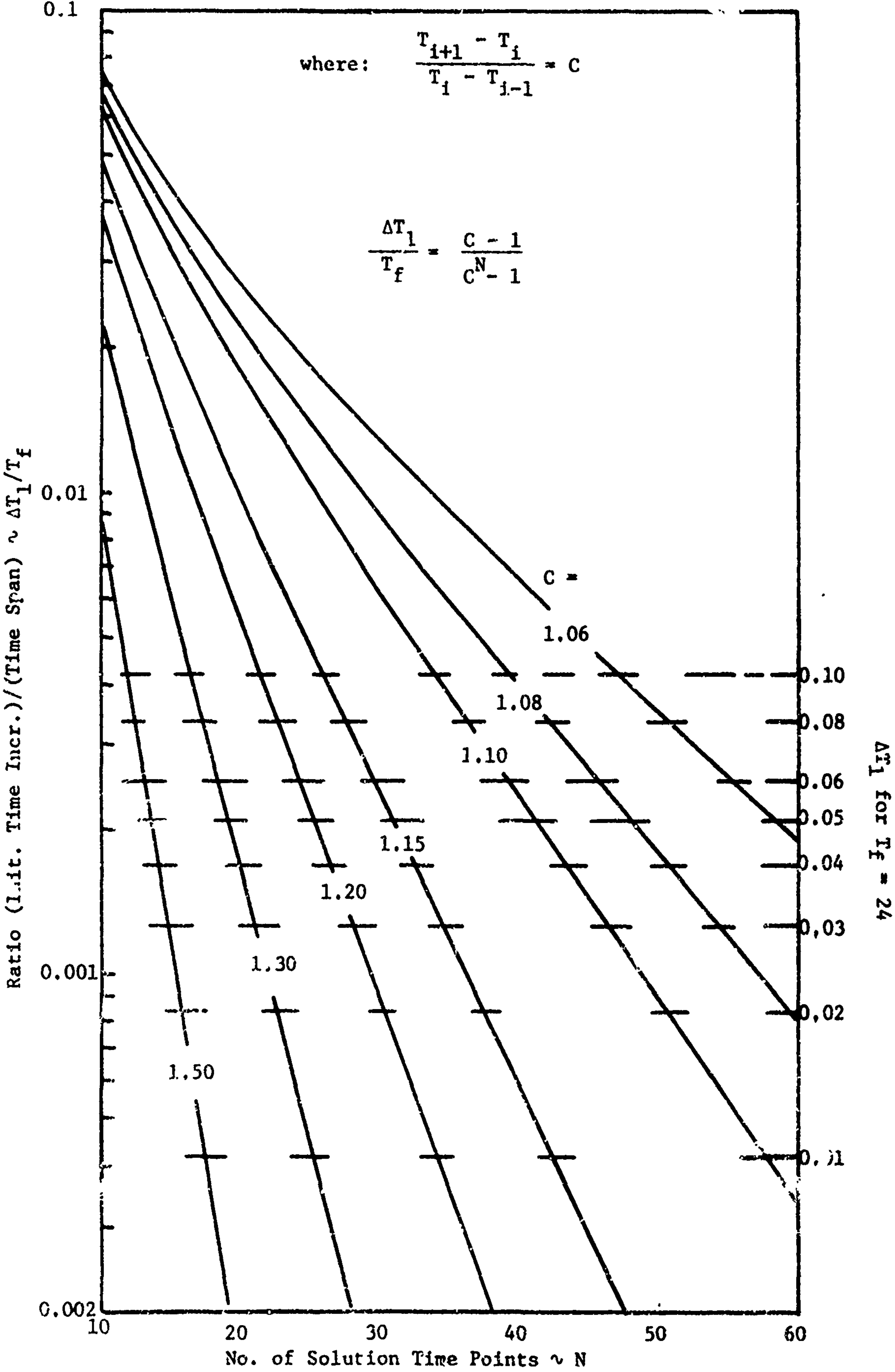


Figure H-1

FINAL SOLUTION TIME INCREMENT

where: $$\frac{T_{i+1} - T_i}{T_i - T_{i-1}} = C$$

$$\frac{\Delta T_N}{T_f} = \frac{C^{N-1}(C-1)}{C^N - 1}$$

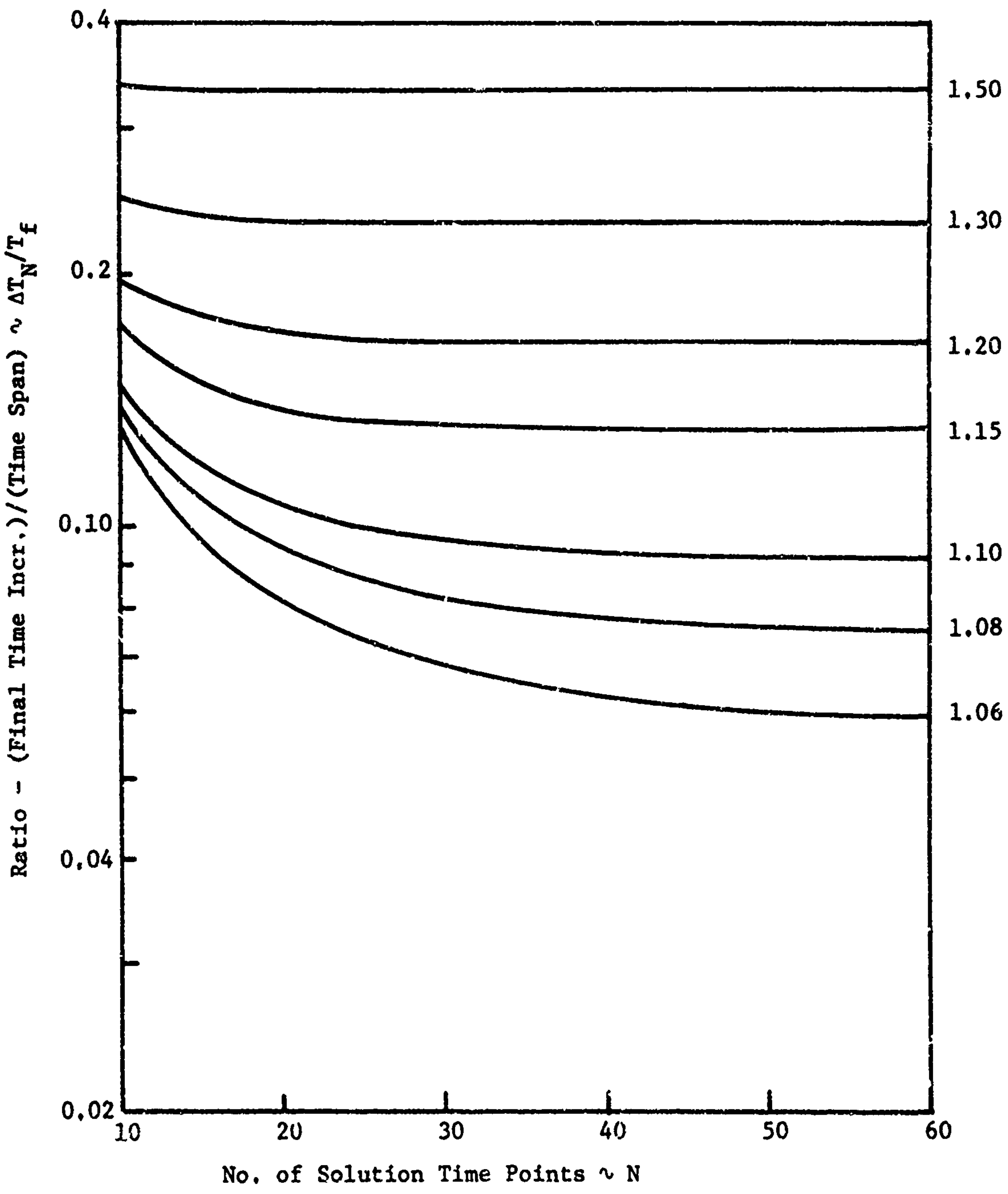


Figure H-2

c. Boundary Condition Function Cards

Skip this section if NBCF = 0

(1) Card One (2I5)

cc		
1-5	N	Number assigned to this function
6-10	NPTS(N)	Number of points in the table describing this function. ≤ 30

(2) Card Two (8F10.0)

cc		
1-10	TFN (N, 1)	Time at point 1, t_1 (T)
11-20	FN (N, 1)	Value of the function at time t_1, f_1
21-30	TFN(N, 2)	Time at point 2, t_2 (T)
31-40	FN (N, 2)	Value of the function at time t_2, f_2
41-50	TFN (N,3)	Point 3
51-60	FN (N, 3)	
61-70	TFN(N,4)	Point 4
71-80	FN (N,4)	

(3) Card Three (8F10.0) (If required)

cc		
1-10	TFN (N,5)	Point 5
11-20	FN (N,5)	

... etc.

Use as many cards 2, 3, etc., in this section as are required to enter NPTS(N) pairs of (TFN(N, I), FN(N, I)) which define this function (Number N). There are NBCF sets of cards 1, 2, 3, etc. in this section. Data for a new function begins on a new card 1.

Figure H-3 represents a function which might be used to describe the pressure transient in a motor. The last point in the table must have a value of time which is greater than or equal to the length of solution period (TMF); for this case $16.0 > 14.0$ = TMF (Figure 3.1).

EXAMPLE BOUNDARY CONDITION FUNCTION

I	TFN (3, I)	FN (3, I)
1	0.0	0.0
2	2.0	0.8
3	4.0	1.0
4	5.0	0.7
5	16.0	0.7

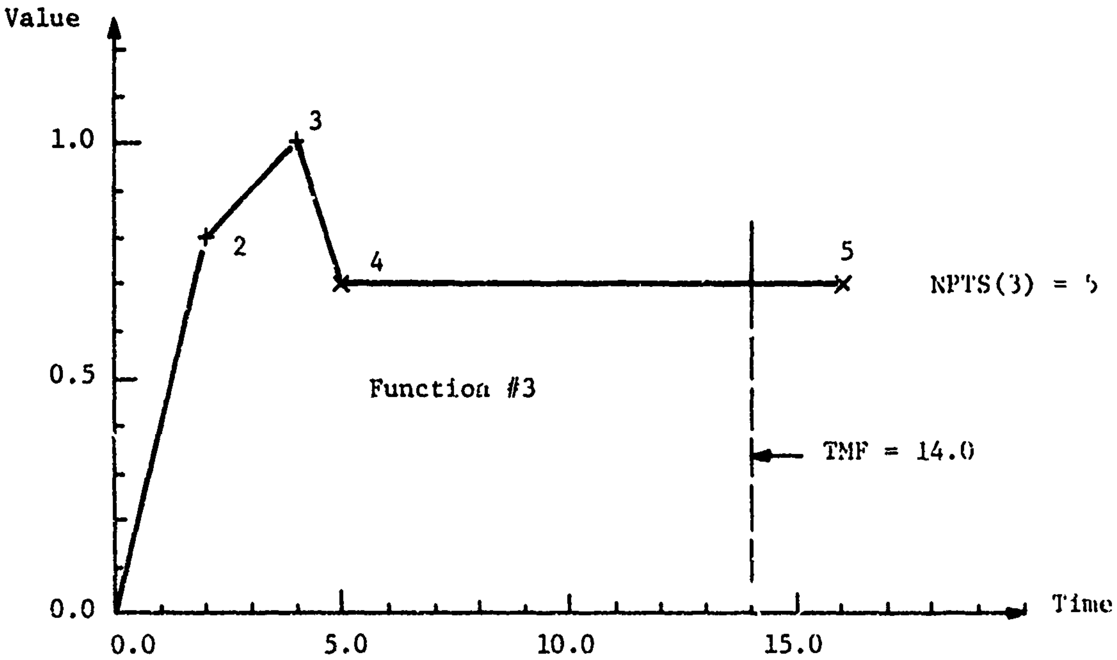


Figure H-3

The table(s) input in this section are used in the prescription of time-dependent boundary conditions such as environmental or nodal point temperatures, nodal point forces and/or displacements, pressure or acceleration loads, etc. One function may be used to describe multiple types of boundary conditions. Values of the functions at times other than TFN (N,I) are calculated within the program using linear interpolation.

If no data are input in this section (NBCF = 0), then all boundary specifications are independent of time and boundary values are assumed to be imposed as step function at $t = 0^+$.

d. Printing Control (16I5) $\leq$ 10 regions

At least one card in this section

Card 1

cc		
1-5	NO(1)	Output Interval
6-10	NPR(1)	Number of print operations at this interval
11-15	NO(2)	Output interval
16-20	NPR(2)	Number of print operations at this interval
	....	
71-75	NO(8)	Output Interval
76-80	NPR(8)	Number of print operations at this interval

Card 2 (If required)

cc		
1-5	NO(9)	Output Interval
6-10	NPR(9)	Number of print operations at this interval
11-15	NO (10)	Output Interval
16-20	NPR (10)	Number of print operations at this interval

This information is used to control the amount of printed output produced by the program. The output scheme defined in this section is used in both the thermal and stress analysis phases. In preparing the data for this block, the following condition must be observed:

$$NO(1) * NPR(1) + NO(2) * NPR(2) + \cdots = NDT$$

This is to say that the printing range must cover the solution period.

Suppose that temperature output is required at time points 2, 4, 7 and 8 of Figure 2.1, then one card would be input in this section:

Card 1: 2,2,3,1,1,1 ; [2 (2) + 3 (1) + 1 (1) = 8].

If temperatures at all 8 points is requested, then

Card 1: 1, 8 ; [1 (8) = 8].

e. Average Element Temperature Cards (3I5, F10.0)

(Card 2-a is the reference for values of NDT, NTEM and TZ).

Skip this section if NTEM = 0, or NTEM = 2

cc		
1-5	NELS	Starting element number for this group.
6-10	NELG	Number of elements with the same average temperature as element number NELS.
11-15	NELI	Element number increment.
16-25	TAVG	Average element temperature for this group of elements. (°F)

If NTEM = 1, there are NDT sets of element temperatures that must be defined in this section (one set for each of the NDT time points contained in the solution period). All element temperatures must be prescribed at a given time point before proceding to the next point. If every element has a different temperature, then NEL cards (with cc 6-15 blank) must be prepared for that time point. It is possible to generate element temperatures at a time point if several elements are at the same temperature. NELG elements are assigned average temperatures TAVG. The number assigned to the first generated element is NELS + NELI; the second is NELS + 2 (NELI), etc.

Suppose that a body described using 50 elements (NEL = 50) is at a uniform 77°F, when $t \leq 0$ (TZ = 77.0), and at the end of the first time increment elements 1-25 drop to 60°F while elements 26-50 reach 45°F. Two cards are required to define the element temperature distribution at the end of time increment one ($t = t_1$):

Card 1: 1, 24, 1, 60.0

Card 2: 26, 24, 1, 45.0

If NTEM = 3, then there is only one set of element temperatures which must be input in this section. This temperature distribution applied to all solution time points; the body starts at $t = 0^-$ at a uniform temperature TZ (at which the body is assumed to be "stress free"), and at $t = 0^+$ the element temperatures assume the values prescribed in this section of the data input and remain constant for all time, $t > 0$.

3. Transient Heat Transfer Solution Data

Skip this section if NTEM = 1, 2, or 3

a. Control Card (One card; 4I5)

cc		
1-5	NMAT	Number of materials with different thermal properties. ≤ 10
6-10	NNBC	Number of nodal point boundary conditions (temperatures or heat fluxes)
11-15	NCBC	Number of convection boundary conditions ≤ 65
16-20	JOB	Job Control Flag = 0 : run heat transfer and use the results to perform the stress analysis. = 1 : run heat transfer, save element temperatures on a permanent file, and use the results to perform the stress analysis. = 2 : run heat transfer, save the results on a permanent file, and stop. = 3 : run heat transfer and stop.
21-25	KAT	KAT = 0 Axisymmetric Analysis KAT $\neq$ 0 Planar Analysis

b. Material Cards (I10, 6F10.0)

cc		
1-10	N	Material number
11-20	XCOND(N)	Conductivity: K_{rr}, $(Btu)(T)^{-1}(L)^{-2}$ (°F/L)
21-30	YCOND(N)	Conductivity: K_{zz}

31-40	XYCOND(N)	Conductivity: K_{rz}
41-50	SPHT (N)	Specific Heat, $(\mathrm{Btu})(\mathrm{F})^{-1}(°\mathrm{F})^{-1}$
51-60	DENS(N)	Weight density, $(\mathrm{F})\ (\mathrm{L})^{-3}$
61-70	QX(N)	Heat generated per unit volume, $(\mathrm{Btu})(\mathrm{T})^{-1}(\mathrm{L})$

Use one card for each different material number assigned in the element array (cards 1-d); NMAT cards must be prepared in this section - order is unimportant.

If the thermal conductivity of a material is independent of direction, then

$$K_{rr} = K_{zz} = k$$

$$k_{rz} = 0$$

The heat generated per unit volume is assumed to be constant with time.

c. Nodal Point Boundary Conditions (2I5, F10.0, I5)

Skip this section if NNBC = 0

cc		
1-5	N	Node Number
6-10	KODE(N)	Boundary condition type
		= 0, externally supplied heat flux = 1, prescribed node temperature
11-20	T(N)	Boundary value amplitude
		= Heat flux (KODE(N) = 0), $(\mathrm{Btu})(\mathrm{T})^{-1}$ (R) = Temperature (KODE(N) = 1), (°F)
21-25	NFN(N)	Function number

All nodal points not specified in this section are assumed to have externally supplied heat flux of zero for all values of time.

A function number equal to zero (or blank) means that the prescribed boundary condition is applied at time zero and remains constant for all time, t >0.

The functions assigned in this section must have been defined previously in Section 2-c. For time varying boundary conditions, the magnitude of the boundary value at some time t is found by selecting the value of the function at t and then multiplying this value times the boundary value amplitude (cc 11-20). A given function can be used to describe any number of boundary conditions.

d. Convection Boundary Conditions

Skip this section if NCBC = 0

(NCBC Cards, 2I5, 2F10.0, I5)

cc		
1-5	I(N)	Node number i
6-10	J(N)	Node number j
11-20	H(N)	Heat transfer coefficient,h: $(BTU)(T)^{-1}(L)^{-2}\ (°F)^{-1}$ $q = h\ (T - T_o)$
21-30	TE(N)	Environmental temperature amplitude, $\bar{T}_o$
31-35	NFCV(N)	Function number

If the environmental temperature T_o is time dependent, then a non-zero function numer must be specified in cc 31-35. $\bar{T}_o$ will be mutliplied by the appropriate value of the function of time t in order to establish the value of environmental temperature, T_o.

If the environment does not change temperature with time, then NFCV(N) = 0 and $T_o = T_o$, constant for t > 0.

If JOB = 2 or 3 (Card 3-a), then data ends here

4. Stress Analysis Information

The program will not read data in this section if NTEM = 0 (card 2a) and JOB = 2 or 3 (card 3a)

a. Title card (20A4)

cc		
1-8	HED	Any alpha numeric information (printed with the solution)

b. Control Card (5I5, 2(F10.0, I5), 3I5, F10.0)

cc			
1-5	NMAT	Number of materials	≤ 4
6-10	NCMN	Number of elements with material identification numbers which are to be redefined.	
11-15	NBCN	Number of node points for which boundary cards are used	≤ 60
16-20	NPC	Number of pressure cards	≤ 55
21-25	NDMG	Number of elements for damage evaluation	≤ 20
26-35	ANGV	Angular velocity amplitude, $(R)\ (T)^{-1}$	
36-40	NFAV	Function number for ANGV	
41-50	AZZ	Axial acceleration amplitude, $(L)\ (T)^{-2}$	
51-55	NFAZ	Function number for AZZ	
56-60	IDSP	Pressure boundary condition function no. for use of pressurization shift function data	
61-65	IPSC	Geometry type flag IPSC = 0 Axisymmetric analysis = 1 Plane strain analysis = 2 Generalized plane strain analysis = 3 Generalized plane stress analysis	
66-70	NSM	Material No. (if IPSC = 2) of case material	
71-80	SZV	Value of normal stress (if IPSC = 3) $(F)\ (L)^{-2}$	

NMAT is the total number of materials (viscoelastic and elastic).

If the material I.D. numbers assigned to the elements in Section 1-d are appropriate for both the heat transfer and stress analyses, then NCMN is set to zero (or blank). NBCN is a count of nodes at which force and/or displacement boundary values are specified. NPC is the total number of element sides subjected to pressure loads.

IDSP is the boundary condition function number which describes the bulk pressure as a function of time when an isothermal pressurization case is being run. For a thermal analysis leave this field blank. This function will be used to find the pressurization shift function values in the properties determination and also will be used in the damage calculations. When IPSC - 2 a special generalized plane strain analysis will be run. The normal strain will be set equal to the average thermal strain ($\alpha_c \Delta T_c$) in the case which is found by the material number NSM specified in cc 66-70.

c. Material Properties

(1) Control Card (3I5, 4F10.0)

cc			
1-5	K	Material number	
6-10	NON(K)	Number of terms in the Prony series representation of the shear relaxation function.	≤ 16
11-15	NSFP(K)	Number of points in the shift function table.	≤ 16
16-25	APO(K)	Equilibrium shear modulus, $(F)(L)^{-2}$	
26-35	XK(K)	Bulk modulus, $(F)(L)^{-2}$	
36-45	ALP(K)	Linear coefficient of thermal expansion, $(L)(L)^{-1}(°F)^{-1}$	
46-55	DENS(K)	Mass density, $(F)(L)^{-4}(T)^{2}$	

The shear relaxation function is written in the form:

$$\phi(t) = A_o + \sum_{i=1}^{M} A_i e^{-\beta_i t}$$

where

$NON(K) = M$ for the Kth material
$APO(K) = A_o$ for the Kth material

An elastic material is input by leaving cc 6-15 blank and entering the shear and bulk moduli in cc 16-25 and cc 26-35, respectively. A non-zero value of density is required for the calculation of body forces arising from specified values of spin velocity and/or axial acceleration.

(2) Prony Series Coefficients Card(s) (8F10.0)

Skip this section if NON(K) = 0

cc			
1-10	AP(K, 1)	A_1 for material K	$(F)(L)^{-2}$
11-20	BP(K, 1)	β_1	$(T)^{-1}$
21-30	AP(K,2)	A_2	$(F)(L)^{-2}$
31-40	BP(K,2)	β_2	$(T)^{-1}$
41-50	AP(K,3)	A_3	$(F)(L)^{-2}$

etc.

Use as many cards in this section as are required to specify NOM(K) pairs of (A_i, β_i); four pairs per card.

(3) Shift Function Table (8F10.0)

Skip this section if NSFP(K) = 0

cc		
1-10	FST(K,1)	Temperature at 1st point, T_1 (°F)
11-20	FS(K,1)	$\text{Log}_{10}\ a_{T_1}$
21-30	FST(K,2)	Temperature at 2nd point, T_2 (°F)
31-40	FS(K,2)	$\text{Log}_{10}\ a_{T_2}$
41-50	FST(K,3)	etc.

Use as many cards in this section as are required to specify NSFP(K) pairs of $(T_i, \log_{10} a_T)$; four pairs per card. If IDSP > 0 (card 4b) FST (K, 1) will be interpreted as pressures and FS (K, 1) are $\text{Log}_{10}\ a_P$.

d. Element Material Numbers

Skip this section if NCMN = 0

cc		
1-5	N1 (1)	Element number
6-10	N2 (1)	Material number assigned to element N1 (1)
11-15	N3 (1)	Number of elements with the same material number as element N1 (1)
16-20	N4 (1)	Element number increment
21-25	N1 (2)	Element number
26-30	N2 (3)	Material number assigned to element N1 (2)
31-35	N3 (2)	Number of elements with the same material number as element N1 (2)
		etc.

Use as many cards in this section as are required to re-define the material numbers of NCMN elements; 16 entries per card are possible.

Suppose that a 50 element problem is to have all its material numbers changed (NCMN = 50), and all even numbered elements are material 1 while all odd elements are material 2; one card describing these changes would read:

[1, 2, 24, 2, 2, 1, 24, 2]

e. Node Point Boundary Specification(s)

At least one card in this section

(I5, 2(I5, F10.0, I5))

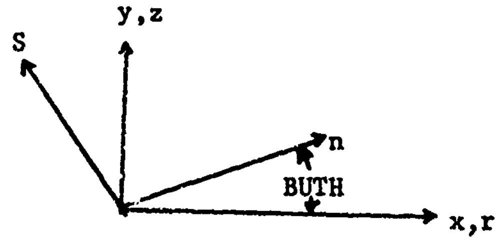
cc			
1-5	K	Node point number (NB(N) = K)	
6-10	NFLR(N)	Radial boundary condition type	
		= 0; externally applied force	(F) $(R)^{-1}$
		= 1; specified displacement	(L)
11-20	BVR(N)	Radial boundary value amplitude	
21-25	NFNR(N)	Function Number	
26-30	NFLZ(N)	Axial boundary condition type	
		= 0; externally applied force	(F) $(R)^{-1}$
		= 1; specified displacement	(L)
31-40	BVZ(N)	Axial boundary value amplitude	
41-45	NFNZ(N)	Function number	
46-55	BUTH(N)	Skew boundary angle (degrees)	

A total of NBCN cards must be prepared in this section. The axial displacement at one node must be specified as a minimum requirement.

Positive boundary values are in the direction of the positive coordinate axes.

Zero (or blank) function numers assigned to boundary value components implies time independence (constant for t > 0). The variation of a boundary value with time is determined by multiplying amplitude times the appropriate value of the corresponding function. All nodes not specified in this section are assumed to have no externally applied loads and are free to displace as the solution dictates.

The shew boundary is shown in the figure below. If BUTH(N) ≠ 0 the boundary conditions are expressed in the n-s system.



f. Pressure Loads

Skip this section if NPC = 0

cc		
1-5	IPC(N)	Node number i
6-10	JPC(N)	Node number j
11-20	PR(N)	Pressure amplitude (F) $(L)^{-2}$
21-25	NFNP(N)	Function

There are NPC cards in this section. NFNP(N) = 0 means that pressure is applied as a step function at $t = 0^+$.

Positive pressure acts in the direction shown in Figure 4.1.

g. Damage Parameters

Skip this section if NDMG = 0

cc			
1-5	LDMG(N)	Element no. whose damage is to be evaluated	
6-15	STZR(N)	σ_{to}	See below
16-25	TZR(N)	t_o	
26-35	SCR(N)	σ_{cr}	

There are NDMG cards in this section. LDMG(N) may be positive or negative. If positive, the hoop stress will be used in damage calculations; if negative the maximum principal r-z stress will be used. The damage is evaluated using

$$P\Sigma D_t = \frac{1}{a_T} \int_0^t \frac{(\sigma(t') - \sigma_{cr} + P(t'))^B}{(\sigma_{to} - \sigma_{cr})^B \; a_p \, (P(t'))} \, dt$$

If IDSP = 0 (card 4b) then P(t') = 0 for all times. If IDSP > 0 the P(t') will be found from the boundary condition function indicated by IDSP.

Data input ends at this point

IV. PROGRAM OUTPUT DESCRIPTION

Output from the program includes:

1. Input geometry, material properties and solution time information in a self-explanatory format.

2. Nodal point temperatures at the specified time-point interval.

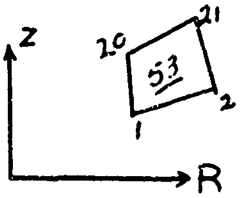
3. The radial, tangential, axial and shear stresses and strains and the average element temperature at the specified time-print interval.

4. The damage rate and accumulated damage for specific elements at each time-print interval.

The printed program output consists of (1) "echo" reproduction of the data input cards identified in self-explanatory format, and (2) results of the analysis which might include nodal point temperatures, element stresses/strains, node displacements, etc.

A. ELEMENT NODE NUMBERING

It should be noted that element node number data may not be listed in the same order as these data appear on the input card. The program logic permutes the order of the element node numbers so that the largest node number is always last (4th) in the printed list for each element. For example, if element number 53 (shown below) is input as (1, 2, 21, 20), the program will print the data as (20, 1, 2, 21) so that "21" is last while the original counter clockwise order is preserved. Efficiency is gained if the user specifies the 4th node as the largest for every element.

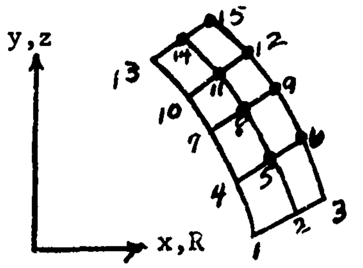


The reason for having the largest node as last one in the sequence for a given element is that three (3) equations are assigned to this node as opposed to only two (2) equations per node for the others. A node number that is not the largest one in any element has the R and Z displacement components as its unknowns (2 total). All other nodes have R, Z displacement components plus the "mean pressure variable" H as unknowns at that point (3 total). Thus, node "21" of element"53" above has assigned to it U_{R21}, U_{Z21}, and H_{53} as unknowns.

In order to assign equations uniquely to the unknown values of "H" for every element, the program has the following restriction:

> The finite element quadrilateral mesh must be numbered so that any node number is maximum in only one (1) element.

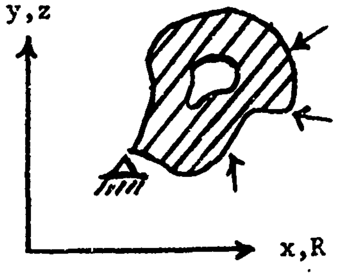
With a "layered" numbering scheme (across the "narrow" direction of the grid) the restriction is complied with automatically:



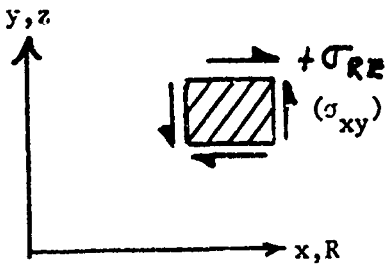
The "heavy" nodes (●) shown above have three (3) unknowns associated with them.

B. SIGN CONVENTION

All positive vector quantities (forces/displacements) are in the directions of the positive coordinate axes providing the grid is positioned in the "positive" R-Z quadrant:



Normal stress (strains) are tensile when positive and the shear stress (strain) sign convention is shown below:



$$PED_t = \frac{1}{a_t} \int_0^t \frac{(\sigma(t') - \sigma_{cr} + P(t'))^B}{(\sigma_{to} - \sigma_{cr})^B \, a_p \, (P(t'))} dt'$$

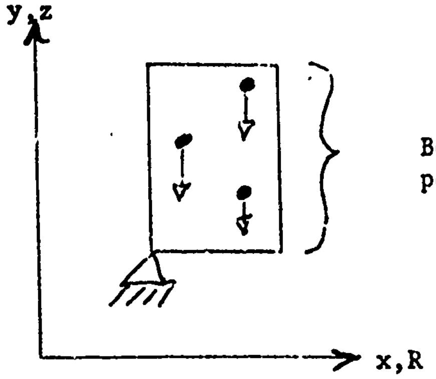
If IDSP = 0 (Card 4b) then P(t') = 0 for all times. If IDSP > 0 the P(t') will be found from the boundary condition function indicated by IDSP.

The value of P(t') is that portion of the inner-bore firing pressure, $P_i(t)$, which is transmitted to the point in the grain where the principal stress, $\sigma(t')$, is evaluated. The relation of P(t') to $P_i(t')$ in infinite-length cylinders is well known( ) (See Section IV of this report). For finite length cylinders the ratio of P(t') to $P_i(t\ )$ must be obtained from the stress analysis.

For metal cases P(t') seldom differs from $P_i(t')$ by more than 5%. For most firing problems this difference is not significant and P(t') can be approximated by $P_i(t')$.

Data input ends at this point

A positive spin velocity produces hoop tension, and positive axial acceleration causes body forces to be applied on the body acting in the (-Z) direction:



V. EXAMPLE PROBLEMS

A. TRANSIENT HEAT CONDUCTION IN A LONG CYLINDER

The purpose of this example is to illustrate the use of the program in solving heat conduction problems. A long, hollow cylinder constructed from a single material is initially at a uniform temperature of 0°F; then, at $t = 0^+$ the outer surface of the cylinder is instantaneously heated to 1°F along its entire length. The ends of the cylinder are insulated against axial heat flow so that at any axial station the heat flow is purely radial.

Figure H-4 shows a one (1) inch slice of the cylinder whose inner and outer radii are 1.00" and 2.00", respectively. The slice has been modeled with a mesh composed of ten (10) equal sized elements; the "Z" axis is the centerline of the cylinder. At time zero the temperature of nodes 11 and 22 is changed from 0°F (TZ = 0.0) to 1°F, and as time proceeds the interior of the cylinder begins to warm up; thermal equilibrium is reached when the entire cylinder is at a uniform 1°F. Nodes 1-10 and 12-21 are insulated in the sense that no externally supplied heat enters the body at these points. The program assumes that all nodes not specifically included as boundary condition nodes are insulated against externally supplied heat flow; i.e., for all non-boundary nodes, the amount of heat entering a node must balance the amount of heat leaving that node in a unit of time.

The solution span is sub-divided into fifty (50) equal time increments of 0.01 hours each so that the time at the end of solution is 0.050 hours (TMF = 0.05), Figure H-5.

SIGN CONVENTION FOR PRESSURE LOADS

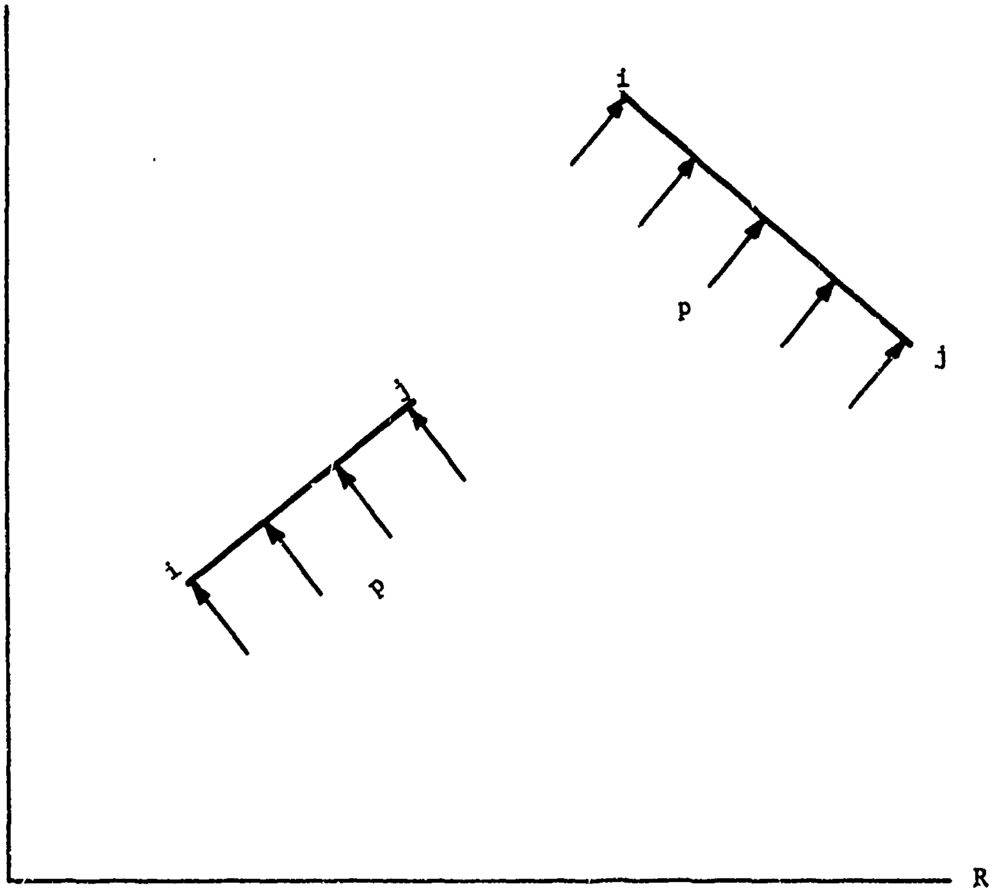
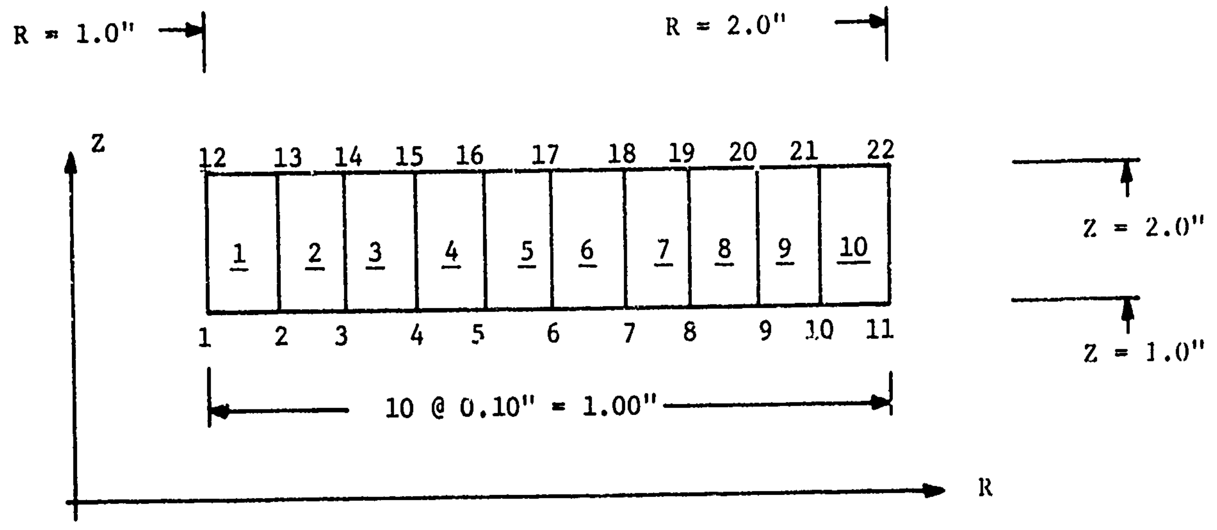
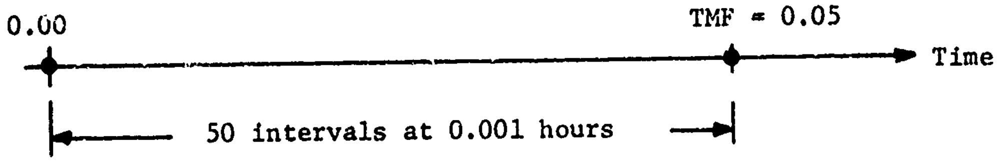


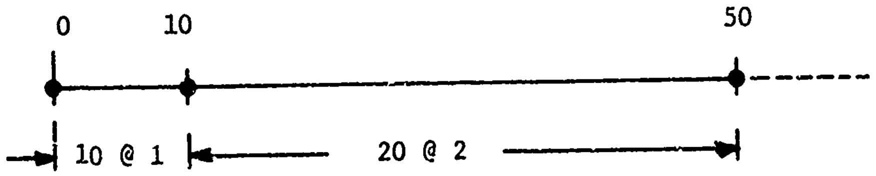
Figure H-4


R = 1.0"
R = 2.0"
Z
12 13 14 15 16 17 18 19 20 21 22
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10 11
Z = 2.0"
Z = 1.0"
10 @ 0.10" = 1.00"
R

a. Mesh For Example Problem A


0.00
TMF = 0.05
Time
50 intervals at 0.001 hours

a. Solution Time Point Array


0
10
50
10 @ 1
20 @ 2

b. Solution Output Intervals

Figure H-5: Solution Time Point and Output Schedules

The output printing schedule is set up so that results are printed at every solution time point for the first ten (10) increments and then at every other solution for the remaining forty (40) increments, Figure H-5.

10 print operations at an interval of 1 = 10
20 print operations at an interval of 2 = $\underline{40}$
50 increments

The thermal properties of the material are summarized as follows:

$K_{rr} = 0.20\ (\text{Btu})(\text{hr.})^{-1}(\text{in.})^{-2}(°\text{F/in.})^{-1}$

$K_{zz} = 0.20$

$K_{rz} = 0.$

Specific Heat = 0.20 (Btu) $(\text{lb.})^{-1}\ (°\text{F})^{-1}$

Weight Density = 0.20 (lb.) $(\text{in.})^{-3}$

Heat generated per unit volume per unit time = 0.[*]

The data cards for this job are shown in Table H-3.

Figure H-6 is a plot of temperature versus time for three (3) points in the cylinder: (1) outer surface (nodes 11 or 22); (2) mid-radius (nodes 6 or 17); and, (3) inner-radius (nodes 1 or 12). For long times, all nodes approach 1°F in the limit.

A value of 3 was assigned to the control variable "JOB" causing a termination of execution after the heat transfer solution. If the results were to be saved[**], then the node temperatures would be averaged for each element before saving the results.

B. ELASTIC RING SUBJECTED TO AXIAL TENSION

The purpose of this example is to illustrate the use of the program in solving elastic problems. A hollow, short cylinder is subjected to an axial tension of 6000 psi on one surface and restrained (without radial shear) against axial displacement on the opposite end (Figure H-7). The problem has been modeled with four (4) quadrilateral elements as shown in Figure H-8. The applied stress has been converted to equivalent concentrated loads of 4000, 12000, and 8000 lbs./radian applied at nodes 7, 8 and 9, respectively.

Output from the computer program is shown in Figure H-9 a, b and c.

---

[*] For purposes of data preparation, unit heat generation is treated as a physical property which can vary from material to material.

[**] JOB = 1 or 2

DATA CARDS FOR EXAMPLE A

360 FORTRAN CODING FORM

USE 360 SYMBOLS

CODER________ DATE________

JOB______ DECK____ SEQ____

COL 73-75 · 76 · 77-78

FORTRAN STATEMENT

```
TVA
EXAMPLE A, HEAT CONDUCTION ANALYSIS OF A CYLINDER WITH STEP BC CHANGE
   22   10    0
    1       1.0       1.0   11       2.0       1.0    1
   12       1.0       2.0   22       2.0       2.0    1
    1    1    2   13   12    1    9    1
   50    1      0.05    0    0       0.0
   50     0.001
    1   10    2   20
    1    2    0    3
         1       0.2       0.2       0.0       0.2       0.2       0.0
   11    1       1.0    0
   22    1       1.0    0
```

Table H-3

TRANSIENT TEMPERATURES IN THE CYLINDER OF EXAMPLE A

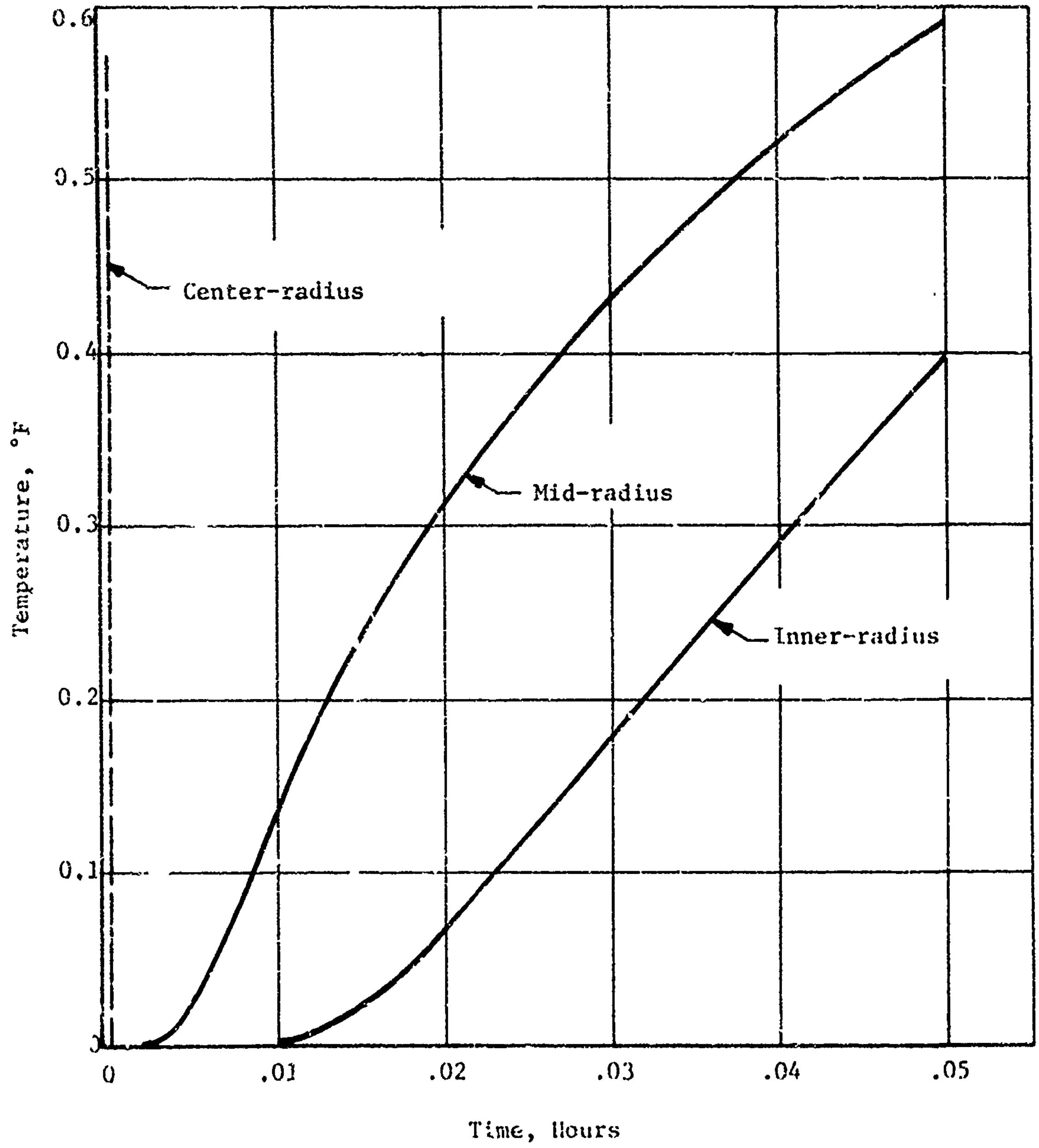


Figure H-6

EXAMPLE B: AXIAL LOADING OF AN ELASTIC CYLINDER

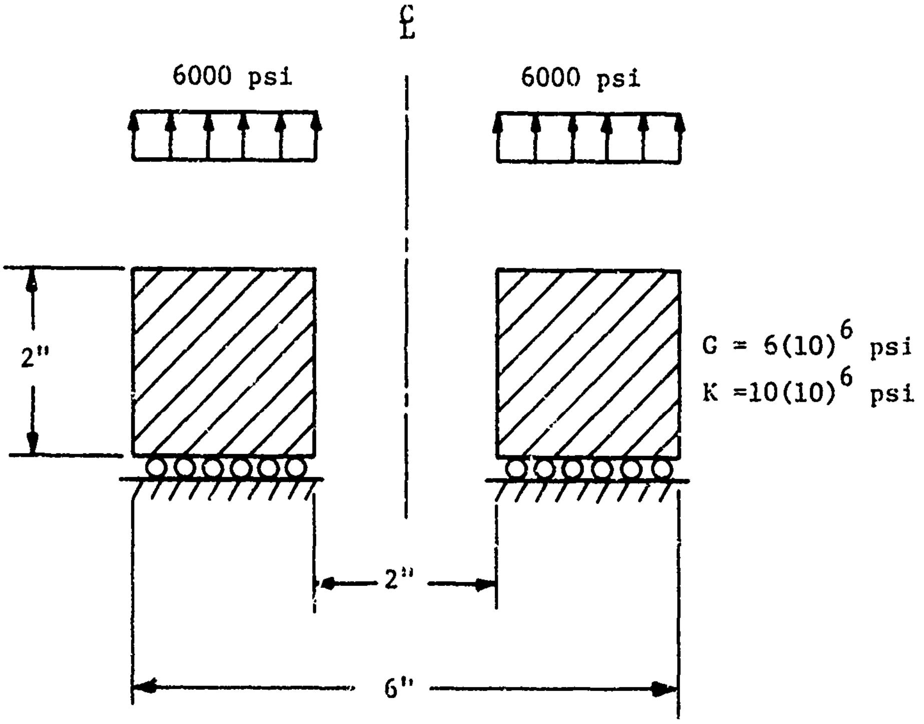

℄
6000 psi
6000 psi
2"
$G = 6(10)^6$ psi
$K = 10(10)^6$ psi
2"
6"

Figure H-7

GRID FOR EXAMPLE B

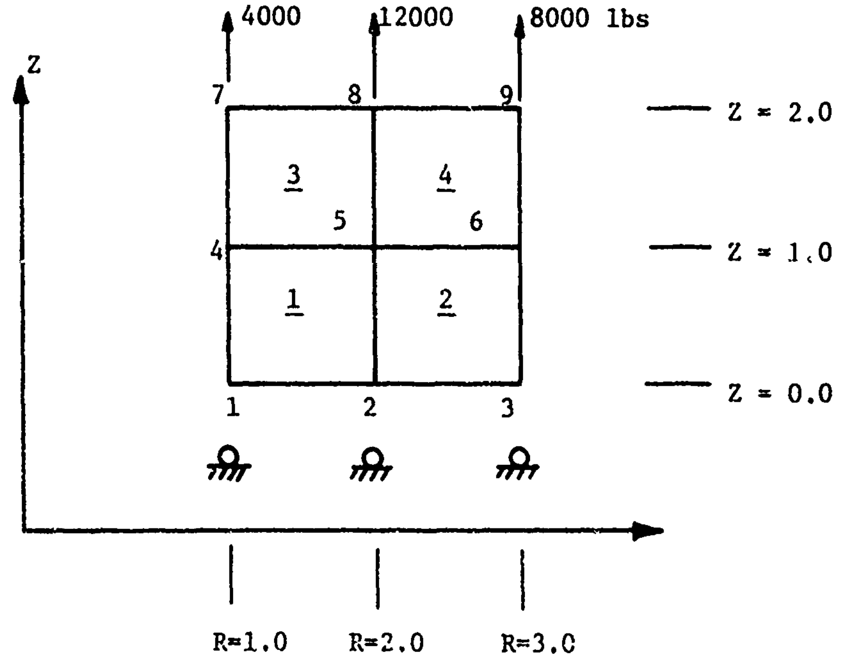

4000
12000
8000 lbs
Z
7
8
9
3
4
5
6
4
1
2
1
2
3
Z = 2.0
Z = 1.0
Z = 0.0
R=1.0
R=2.0
R=3.0

Figure H-8

```
TWO-DIMENSIONAL THERMOVISCOELASTIC ANALYSIS
  R/A, DIRECT-AXIAL-LOADING  (LRH SAMPLE)  (HOLLOW CYLINDER)       LRH-1.1
CONTROL PARAMETERS

     NUMBER OF NODES .......      9
     NUMBER OF ELEMENTS ...       4


NODE COORDINATE DATA

     NODE  R-COORDINATE  Z-COORDINATE

       1      1.00000        0.0
       2      2.00000        0.0
       3      3.00000        0.0
       4      1.00000        1.00000
       5      2.00000        1.00000
       6      3.00000        1.00000
       7      1.00000        2.00000
       8      2.00000        2.00000
       9      3.00000        2.00000


ELEMENT DATA

     ELEMENT    NODE 1    NODE 2    NODE 3    NODE 4    MATERIAL

        1          4         1         2         5          1
        2          5         2         3         6          1
        3          7         4         5         8          1
        4          8         5         6         9          1


TIME - TEMPERATURE CONTROL INFORMATION

     TOTAL NUMBER OF TIME INCREMENTS.........................          1
     NUMBER OF REGIONS WITH EQUAL TIME INCREMENTS............          1
     TIME AT END OF LAST INCREMENT...........................       60.000
     NUMBER OF TIME DEPENDENT BOUNDARY CONDITION FUNCTIONS..           0
     ELEMENT TEMPERATURE FLAG................................          3
     STRESS-FREE TEMPERATURE.................................          0.0


SOLUTION TIME POINTS

     REGION    NUMBER OF     INCREMENT
              INCREMENTS       VALUE

        1          1          60.0000


ELEMENT TEMPERATURE PRINT CONTROL

     OUTPUT INTERVAL       PRINT OPERATIONS

             1                     1


AVERAGE ELEMENT TEMPERATURES
(ELEMENT TEMPERATURE DISTRIBUTION CONSTANT)
     SOLUTION TIME VALUE =   0.60000E 02

     ELEMENT     AVERAGE TEMPERATURE

        1              0.0
        2              0.0
        3              0.0
        4              0.0
```

Figure H-9a

```
MECHANICAL PROPERTIES INFORMATION

 P/A = 6000.0 PSI, AXIAL


       NUMBER OF MATERIALS.................................        1
       NUMBER OF ELEMENTS WITH REDEFINED MATERIALS.......         0
       NUMBER OF NODES WITH SPECIFIED BOUNDARY VALUES...          8
       NUMBER OF PRESSURE CARDS..........................         0
       ANGULAR VELOCITY AMPLITUDE..........          0.0
       ANGULAR VELOCITY FUNCTION...........          0
       AXIAL ACCELERATION AMPLITUDE........          0.0
       AXIAL ACCELERATION FUNCTION.........          0
       OUTPUT INTERVAL FLAG................          0
       DISPLACEMENT PRINT CONTROL FLAG.....          0


 MATERIALS DATA

 MATERIAL         TERMS IN           POINTS IN    EQUILIBRIUM            BULK       EXPANSI
  NUMBER      PRONY SERIES     SHIFT FUNCTION         MODULUS         MODULUS     COEFFICIE

      1                0                   .0    6000000.000    10000000.000     0.0


SERIES REPRESENTATION OF THE SHEAR RELAXATION FUNCTIONS

MATERIAL          TERM IN     COEFFICIENT          EXPONENT
 NUMBER        THE SERIES        ALPHA(I)           BETA(I)

     1       ELASTIC MATERIAL (NO SERIES EXPANSION)


TABULATION OF TIME-TEMPERATURE SHIFT FUNCTIONS

MATERIAL      POINT      TEMPERATURE       SHIFT FACTOR
 NUMBER      NUMBER             T(I)    LOG-10 A(T(I))

     1        TEMPERATURE INDEPENDENT MATERIAL


NODAL POINT BOUNDARY CONDITIONS

                        R A D I A L                                      A X I A L
   NODE          TYPE             BOUNDARY     FUNCTION         TYPE         BOUNDARY        FUNCTION
 NUMBER          CODE                VALUE       NUMBER         CODE            VALUE          NUMBER

      1             0     0.0                         0            1     0.0                          0
      2             0     0.0                         0            1     0.0                          0
      3             0     0.0                         0            1     0.0                          0
      4             0     0.0                         0            0     0.0                          0
      5             0     0.0                         0            0     0.0                          0
      7             0     0.0                         0            0     0.0                          0
      8             0     0.0                         0            0     0.4000000E 04                0
      9             0     0.0                         0            0     0.1200000E 05                0
                                                                         0.8000000E 04                0


NODE          DISPLACEMENT                     TIME=      6.000000E 01
  1    -1.00000E-04      0.0
  2    -2.00000E-04      0.0
  3    -3.00001E-04      0.0
  4    -1.00001E-04      4.00000E-04
  5    -2.00001E-04      4.00001E-04
  6    -3.00002E-04      4.00001E-04
  7    -1.00003E-04      8.00001E-04
  8    -2.00003E-04      8.00003E-04
  9    -3.00003E-04      9.00002E-04
```

NOT REPRODUCIBLE

Figure. H-9b

```
       SIGR            SIGTH           SIGZ          . SIGRZ
1 -1.22070E-03 -7.81250E-03  6.00000E 03 -4.14729E-03
2 -4.39453E-03  1.22070E-03  6.00002E 03 -2.48838E-03
3 -9.76562E-03 -2.73437E-02  6.00001E 03 -4.36557E-05
4 -4.15039E-03 -9.76562E-03  6.00001E 03 -3.97267E-03


        EPR            EPTH            EPZ           EPRZ          TEM
  -9.99998E-05 -1.00000E-04  4.00000E-04 -6.91216E-10  0.0
  -1.00001E-04 -1.00000E-04  4.00002E-04 -4.14730E-10  0.0
  -9.99999E-05 -1.00001E-04  4.00002E-04 -7.27596E-12  0.0
  -1.00000E-04 -1.00001E-04  4.00001E-04 -6.62112E-10  0.0
```

Figure H-9c

The solution time span which has no meaning when constant loads are applied to elastic media consists of one (1) time increment arbitrarily selected as being 60 seconds long; one print operation is performed at the end of the first (and only) time increment. The average temperature of all elements is read in as 0°F. The pertinent mechanical properties are the shear ($G = 6(10)^6$ psi) and bulk ($K = 10(10)^6$ psi) moduli of the material. There were eight (8) of the nine (9) nodes designated as boundary nodes; nodes 4 and 6 are unloaded and unrestrained and do not have to appear in the statement of boundary conditions. Nodes 4 and 6 were inadvertently retained from a previous data deck.

The results consist of R and Z displacements of each node, average element stresses and strains and the average element temperatures all quoted at $t_1 = 60$ seconds. The applied axial stress is recovered exactly as 6000 psi (SIGZ) constant in each element.

C. PLANE-STRAIN VISCOELASTIC CYLINDER, EXAMPLE PROBLEM

The problem is a long cylindrical bore propellant grain (1.50 I.D., 8.0 O.D.) in a .060 steel case. The element geometry is shown in Figure H-10. The propellant relaxation and shift function curves are shown in Figures H-11 and H-12. Sixteen decade reduced time points were used to perform a prony series curve fit to the relaxation function and the coefficients are shown in Table H-4.

The motor, initially stress free at 135°F, was subjected to the thermal environment, through a heat transfer coefficient at the case, shown in Figure H-13. All nodes in the system were fixed in the axial direction to simulate a plane strain condition and the linear cumulative damage was calculated for two elements in the system, one at bore and one next to the case - grain bond.

A listing of the data input cards required to describe this problem is shown in Table H-5. Line 2 is the title; line 3 is the control card; lines 4-9 describe the node point coordinates; lines 10, 11 establish the element I.D.; line 12 is the solution time point and reference temperature control and lines 13-29 describe the solution time points. Lines 31-36 describe the free stream temperature function. Line 37 describes the output print interval. Lines 39 and 40 are the thermal properties for the grain and case. Line 41 describes the convection boundary condition. Line 42 is the stress analysis title card and 43 is the stress control card. Lines 45-48 are the relaxation series coefficients. Lines 49-51 are the shift function points. Line 52 describes the case properties. Lines 53-92 are the displacement boundary conditions. The last two lines give the damage parameters.

Selected portions of the output from this analysis are shown in Table H-6.

GEOMETRY FOR PLANE STRAIN EXAMPLE

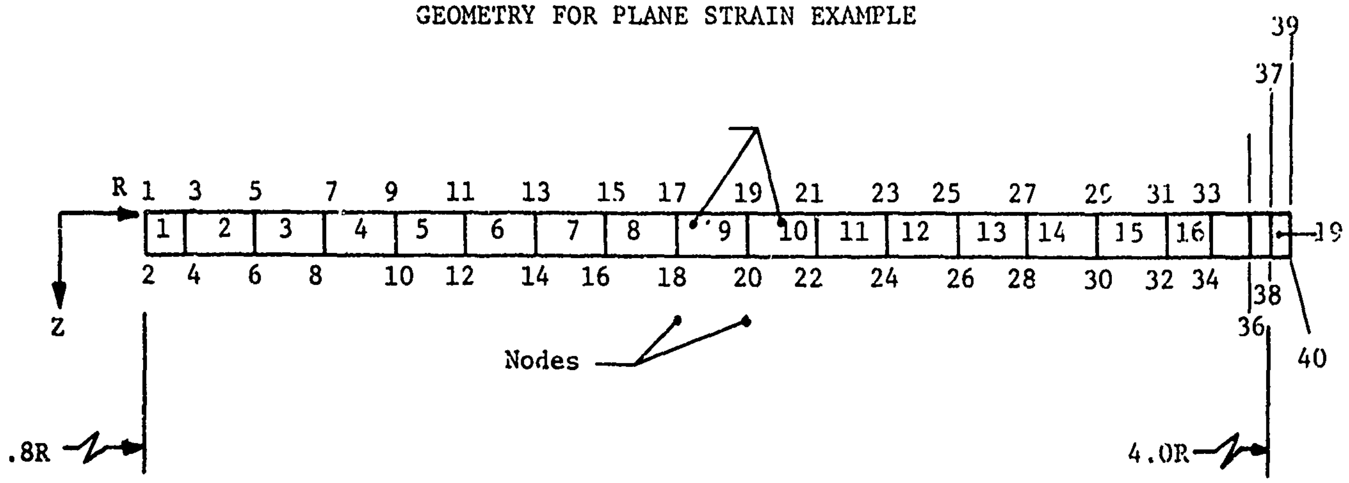

R
Z
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
19
Nodes
.8R
4.0R

Figure H-10

RELAXATION MODULUS OF A CTPB PROPELLANT

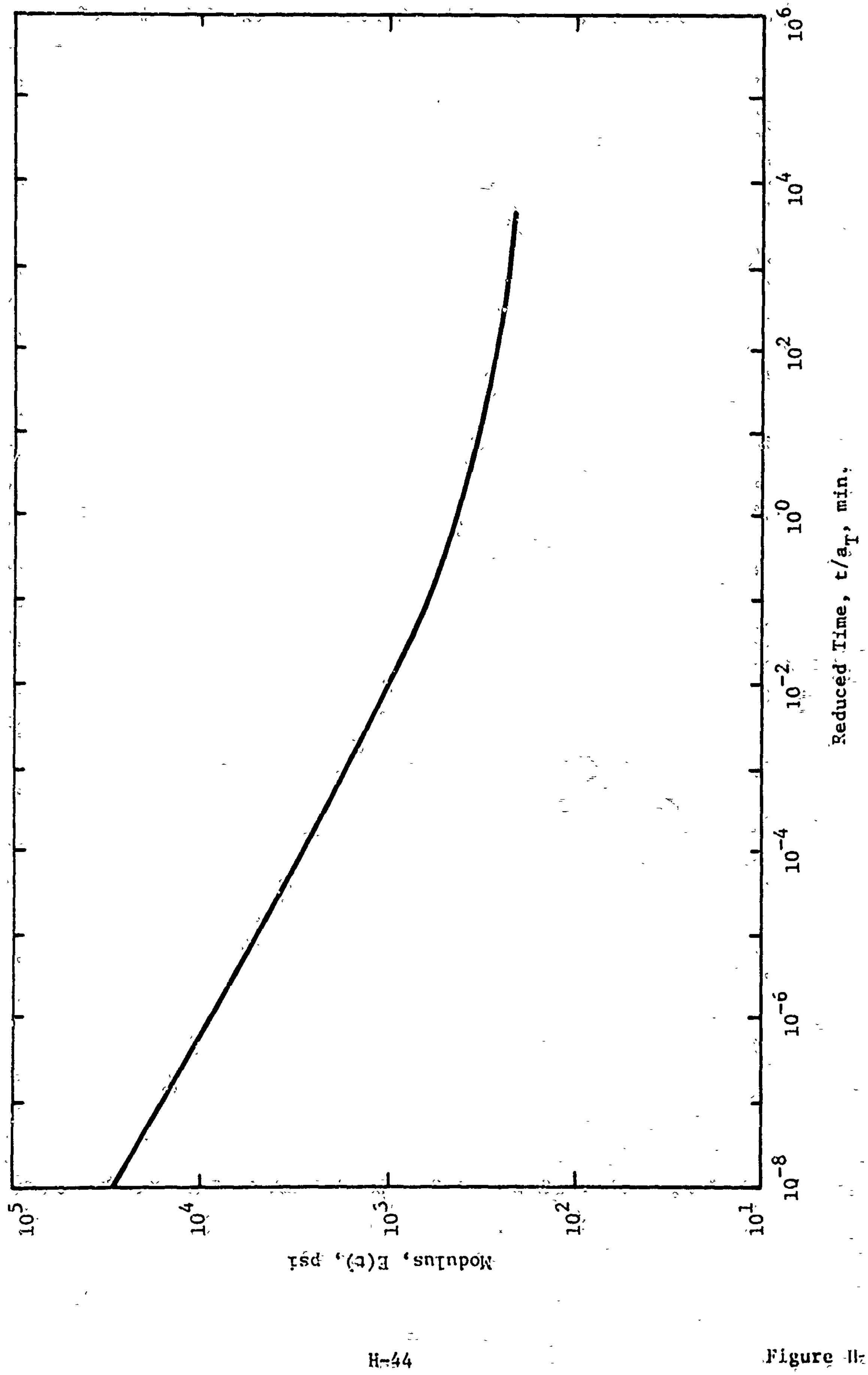


Figure H-11

TIME TEMPERATURE SHIFT FACTORS FOR A CTPB PROPELLANT

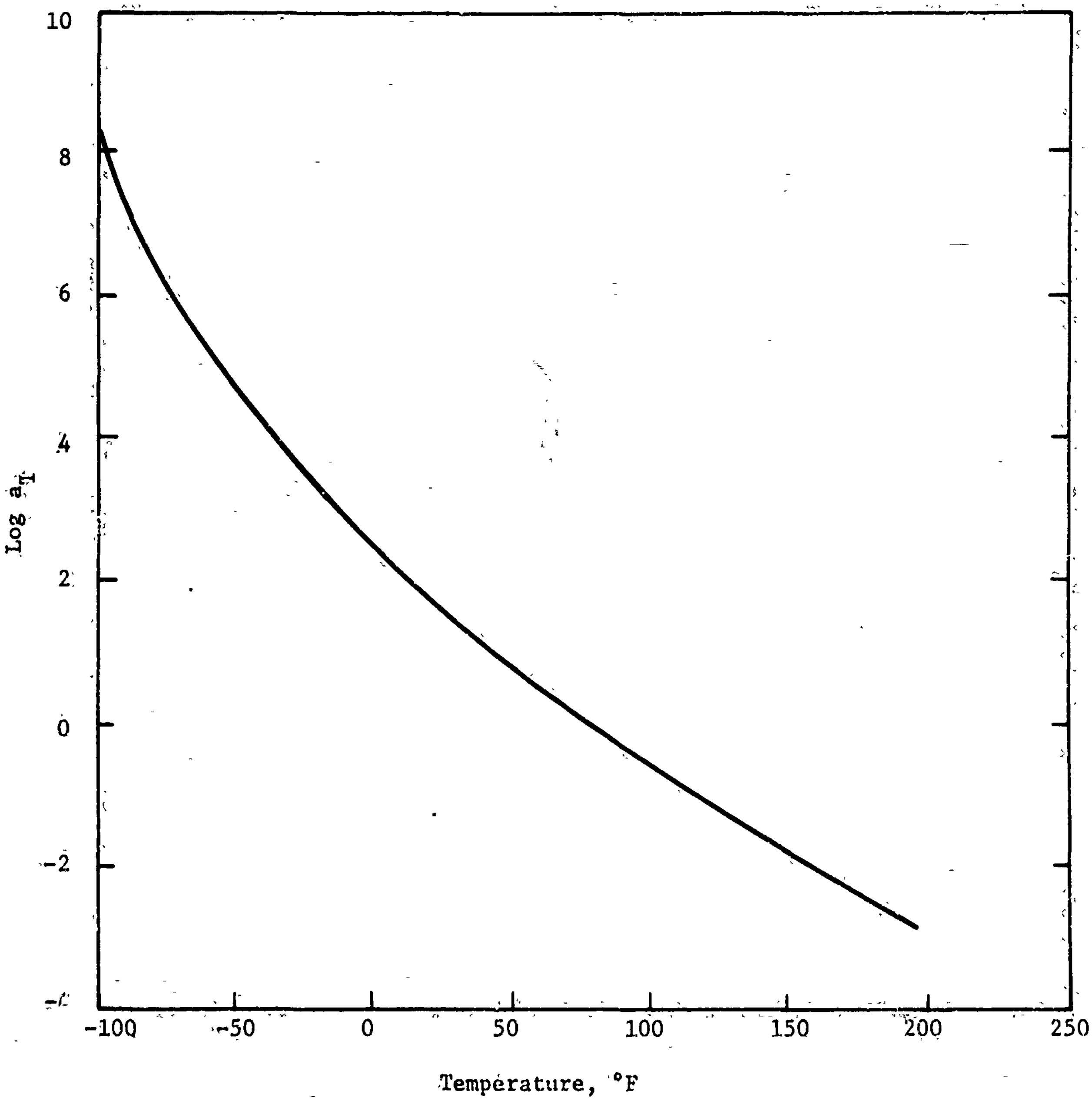


Figure H-12

Propellant Relaxation Function

log(t/a_T) (min)	E_r (psi)	B_i (hr^{-1})	A_i	$A_i/3$	i
			100	33.333	0
-9	44000	3×10^{10}	25836	8612	1
-8	29000	3×10^{9}	12087	4029	2
-7	14000	3×10^{8}	6919.2	2306.4	3
-6	8300	3×10^{7}	3915.5	1305.2	4
-5	5000	3×10^{6}	2435.2	811.40	5
-4	3000	3×10^{5}	1341.9	447.3	6
-3	1800	3×10^{4}	1033.9	344.63	7
-2	1000	3×10^{3}	457.98	152.66	8
-1	620	3×10^{2}	265.80	88.60	9
0	410	3×10^{1}	137.74	45.91	10
1	320	3×10^{6}	.16481	.0546	11
2	280	3×10^{-1}	113.47	37.82	12
3	210	3×10^{-2}	-2.9029	-.9676	13
4	190	3×10^{-3}	60.2506	20.0835	14
5	150	3×10^{-4}	4.26975	1.42325	15
6	130	3×10^{-5}	49.4143	1.64714	16

$$E_r = A_o + \sum_{i=1}^{16} A_i \, e^{-B_i t}$$

FREE-STREAM TEMPERATURE
(H = .0125 BTU/HR/IN.^{2}/F)

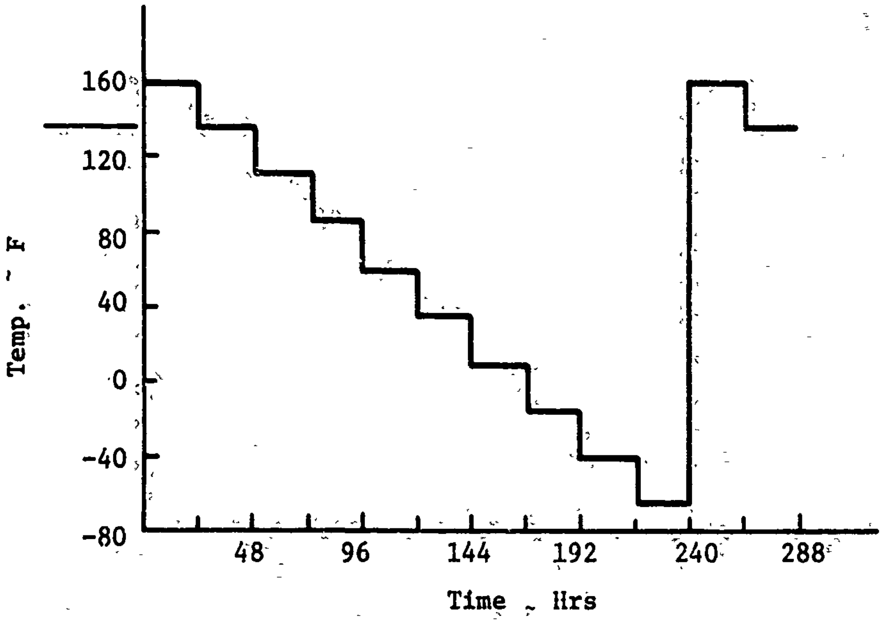


Figure H-13

Data Input for Example Problem 3

```
VISCOELASTIC PARAMETER STUDY B/A=5.00 B=4 EQUIVALENT PLANE STRAIN M=1P=1
   40   19    1
    1         .0        3.   2        .8        .1   1
    3         .9        0.  31       3.8        0.   2
    4         .9        .1  32       3.8        .1   2
   33        3.9        0.  37        4.        0.   2
   34        3.9        .1  38        4.        .1   2
   39       4.00        0.  40      4.00        .1   1
    1    1     3    4    2   1   17   2
   19   37    39   40   38   2
 .336   84        288.   1   0      .135.
    4        .025    4      .055    4       .17    4       .45    4       .8
    4        1.50    4      3.00    4      .025    4      .055    4      .170
    4        .450    4      .800    4      1.50    4      3.00    4      .025
    4        .055    4      .170    4      .450    4      .800    4      1.50
    4        3.00    4      .025    4      .055    4      .170    4      .450
    4        .800    4      1.50    4      3.00    4      .025    4      .055
    4        .170    4      .450    4      .800    4      1.50    4      3.00
    4        .025    4      .055    4       .17    4       .45    4       .8
    4        1.50    4      3.00    4      .025    4      .055    4      .170
    4        .450    4      .800    4      1.50    4      3.00    4      .025
    4        .055    4      .170    4      .450    4      .800    4      1.50
    4        3.00    4      .025    4      .055    4      .170    4      .450
    4        .800    4      1.50    4      3.00    4      .025    4      .055
    4        .170    4      .450    4      .800    4      1.50    4      3.00
    4        .025    4      .035    4       .17    4       .45    4       .8
    4        1.50    4      3.00    4      .025    4      .055    4      .170
    4        .450    4      .800    4      1.50    4      3.00    4      .025
    1   24
   0.005     100.0     24.0     100.0     24.005     135.0     48.0     135.0
  48.005     110.0     72.0     110.0     72.005      85.0     96.0      85.0
  96.005      60.0    120.0      60.0    120.005      35.0    144.0      35.0
 144.005      10.0    168.0      10.0    168.005     -15.0    192.0     -15.0
 192.005     -40.0    216.0     -40.0    216.005     -65.0    240.0     -65.0
 240.005     100.0    264.0     100.0    264.005     135.0    288.0     135.0
    4   84
    2    0      1     0
         1    .02333     .02333       0.       .315      .0633
         2      2.5        2.5        0.       .118      .283
   39   40     .0125        1.     1.
VISCOELASTIC PARAMETER STUDY B/A=5.00 B=4 EQUIVALENT PLANE STRAIN
    2    0     40          2
    1   16     12        33.1       5.E5      3.67E-5
  8612.0        3.0E10 4029.0        3.0E9 2306.4        3.0E8 1305.2        3.0E7
   815.4        3.0E6  447.3         3.0E5  344.63       3.0E4  152.68       3.0E3
    80.6        3.0E2   45.95        3.0E1    0.0546     3.0     37.82       3.0E-1
  -0.9670       3.0E-2  20.0835      3.0E-3   1.42325    3.0E-4   1.04714    3.0E-5
 -100.0     8.28     -75.0     6.00     -50.0     4.00     -25.0     3.54
    0.0     2.52      25.0     1.60      50.0     0.80      77.0     0.0
  100.0    -0.56     125.0    -1.22     150.0    -1.82     200.0    -2.94
    2    0     0     11.5E6     25.E6     5.9E-6
    1                               1
    2                               1
    3                               1
    4                               1
    5                               1
    6                               1
    7                               1
    8                               1
    9                               1
   10                               1
   11                               1
   12                               1
   13                               1
   14                               1
   15                               1
   16                               1
   17                               1
   18                               1
   19                               1
   20                               1
   21                               1
   22                               1
   23                               1
   24                               1
   25                               1
   26                               1
   27                               1
   28                               1
   29                               1
   30                               1
   31                               1
   32                               1
   33                               1
   34                               1
   35                               1
   36                               1
   37                               1
   38                               1
   39                               1
   40                               1
    1         130..016000007             0.       8.43
  -19         130..016000007             0.       8.43
```

Table H-6

Table 5.4

Computer Output for Example Problem C

```
TWO-DIMENSIONAL THERMOVISCOELASTIC ANALYSIS
    VISCOELASTIC PARAMETER STUDY B/A=5.00 B=4 EQUIVALENT PLANE STRAIN M=1P=1
  CONTROL PARAMETERS
        NUMBER OF NODES ......    40
        NUMBER OF ELEMENTS ...    19


  NODE COORDINATE DATA
        NODE  R-COORDINATE  Z-COORDINATE
          1     0.80000     0.0
          2     0.80000     0.10000
          3     0.90000     0.0
          4     0.90000     0.10000
          5     1.10714     0.0
          6     1.10714     0.10000
          7     1.31428     0.0
          8     1.31428     0.10000
          9     1.52143     0.0
         10     1.52143     0.10000
         11     1.72857     0.0
         12     1.72857     0.10000
         13     1.93571     0.0
         14     1.93571     0.10000
         15     2.14285     0.0
         16     2.14285     0.10000
         17     2.34999     0.0
         18     2.34999     0.10000
         19     2.55714     0.0
         20     2.55714     0.10000
         21     2.76428     0.0
         22     2.76428     0.10000
         23     2.97142     0.0
         24     2.97142     0.10000
         25     3.17856     0.0
         26     3.17856     0.10000
         27     3.38570     0.0
         28     3.38570     0.10000
         29     3.59284     0.0
         30     3.59284     0.10000
         31     3.80000     0.0
         32     3.80000     0.10000
         33     3.90000     0.0
         34     3.90000     0.10000
         35     3.95000     0.0
         36     3.95000     0.10000
         37     4.00000     0.0
         38     4.00000     0.10000
         39     4.06000     0.0
         40     4.06000     0.10000
```

```
ELEMENT DATA                                                        Table 5.4 (cont.)

     ELEMENT   NODE 1   NODE 2   NODE 3   NODE 4   MATERIAL
         1        2        1        3        4        1
         2        4        3        5        6        1
         3        6        5        7        8        1
         4        8        7        9       10        1
         5       10        9       11       12        1
         6       12       11       13       14        1
         7       14       13       15       16        1
         8       16       15       17       18        1
         9       18       17       19       20        1
        10       20       19       21       22        1
        11       22       21       23       24        1
        12       24       23       25       26        1
        13       26       25       27       28        1
        14       28       27       29       30        1
        15       30       29       31       32        1
        16       32       31       33       34        1
        17       34       33       35       36        1
        18       36       35       37       38        1
        19       38       37       39       40        2


TIME - TEMPERATURE CONTROL INFORMATION

     TOTAL NUMBER OF TIME INCREMENTS..............................   336
     NUMBER OF REGIONS WITH EQUAL TIME INCREMENTS.................    84
     TIME AT END OF LAST INCREMENT................................   200.000
     NUMBER OF TIME DEPENDENT BOUNDARY CONDITION FUNCTIONS..       1
     ELEMENT TEMPERATURE FLAG.....................................     0
     STRESS-FREE TEMPERATURE......................................   135.000
```

Table H-6 Cont'd

Table 5.4 (cont.)

SOLUTION TIME POINTS

REGION	NUMBER OF INCREMENTS	INCREMENT VALUE
1	4	0.0250
2	4	0.0550
3	4	0.1700
4	4	0.4500
5	4	0.8000
6	4	1.5000
7	4	3.0000
8	4	0.0250
9	4	0.0550
10	4	0.1700
11	4	0.4500
12	4	0.8000
13	4	1.5000
14	4	3.0000
15	4	0.0250
16	4	0.0550
17	4	0.1700
18	4	0.4500
19	4	0.8000
20	4	1.5000
21	4	3.0000
22	4	0.0250
23	4	0.0550
24	4	0.1700
25	4	0.4500
26	4	0.8000
27	4	1.5000
28	4	3.0000
29	4	0.0250
30	4	0.0550
31	4	0.1700
32	4	0.4500
33	4	0.8000
34	4	1.5000
35	4	3.0000
36	4	0.0250
37	4	0.0550
38	4	0.1700
39	4	0.4500
40	4	0.8000
41	4	1.5000
42	4	3.0000
43	4	0.0250
44	4	0.0550
45	4	0.1700
46	4	0.4500
47	4	0.8000
48	4	1.5000
49	4	3.0000
50	4	0.0250
51	4	0.0550
52	4	0.1700
53	4	0.4500
54	4	0.8000
55	4	1.5000
56	4	3.0000
57	4	0.0250
58	4	0.0550
59	4	0.1700
60	4	0.4500
61	4	0.8000
62	4	1.5000
63	4	3.0000
64	4	0.0250
65	4	0.0550
66	4	0.1700
67	4	0.4500
68	4	0.8000
69	4	1.5000
70	4	3.0000
71	4	0.0250
72	4	0.0550
73	4	0.1700
74	4	0.4500
75	4	0.8000
76	4	1.5000
77	4	3.0000
78	4	0.0250
79	4	0.0550
80	4	0.1700
81	4	0.4500
82	4	0.8000
83	4	1.5000
84	4	3.0000

Table H-6 Cont'd

Table 5.4 (cont.)

```
SOLUTION TO THE TRANSIENT HEAT CONDUCTION PROBLEM

TIME= 0.100000 00
  1  0.135000E 03    2  0.135000E 03    3  0.135000E 03    4  0.135000E 03    5  0.135000E 03    6  0.135000E 03
  7  0.135000E 03    8  0.135000E 03    9  0.135000E 03   10  0.135000E 03   11  0.135000E 03   12  0.135000E 03
 13  0.135000E 03   14  0.135000E 03   15  0.135000E 03   16  0.135000E 03   17  0.135000E 03   18  0.135000E 03
 19  0.135002E 03   20  0.135002E 03   21  0.135010E 03   22  0.135010E 03   23  0.135040E 03   24  0.135040E 03
 25  0.135143E 03   26  0.135143E 03   27  0.135425E 03   28  0.135425E 03   29  0.136026E 03   30  0.136026E 03
 31  0.137109E 03   32  0.137109E 03   33  0.137893E 03   34  0.137893E 03   35  0.138341E 03   36  0.138341E 03
 37  0.138748E 03   38  0.138748E 03   39  0.138749E 03   40  0.138749E 03

TIME= 0.320000 00
  1  0.135001E 03    2  0.135001E 03    3  0.135001E 03    4  0.135001E 03    5  0.135002E 03    6  0.135002E 03
  7  0.135006E 03    8  0.135006E 03    9  0.135013E 03   10  0.135013E 03   11  0.135028E 03   12  0.135028E 03
 13  0.135059E 03   14  0.135059E 03   15  0.135116E 03   16  0.135116E 03   17  0.135219E 03   18  0.135219E 03
 19  0.135395E 03   20  0.135395E 03   21  0.135679E 03   22  0.135679E 03   23  0.136113E 03   24  0.136113E 03
 25  0.136742E 03   26  0.136742E 03   27  0.137613E 03   28  0.137613E 03   29  0.138761E 03   30  0.138761E 03
 31  0.140191E 03   32  0.140191E 03   33  0.141012E 03   34  0.141012E 03   35  0.141459E 03   36  0.141459E 03
 37  0.141859E 03   38  0.141859E 03   39  0.141860E 03   40  0.141860E 03

TIME= 0.100000 01
  1  0.135695E 03    2  0.135695E 03    3  0.135704E 03    4  0.135704E 03    5  0.135781E 03    6  0.135781E 03
  7  0.135928E 03    8  0.135928E 03    9  0.136143E 03   10  0.136143E 03   11  0.136432E 03   12  0.136432E 03
 13  0.136801E 03   14  0.136801E 03   15  0.137257E 03   16  0.137257E 03   17  0.137811E 03   18  0.137811E 03
 19  0.138469E 03   20  0.138469E 03   21  0.139239E 03   22  0.139239E 03   23  0.140125E 03   24  0.140125E 03
 25  0.141130E 03   26  0.141130E 03   27  0.142254E 03   28  0.142254E 03   29  0.143495E 03   30  0.143495E 03
 31  0.144829E 03   32  0.144829E 03   33  0.145529E 03   34  0.145529E 03   35  0.145901E 03   36  0.145901E 03
 37  0.146217E 03   38  0.146217E 03   39  0.146217E 03   40  0.146217E 03

TIME= 0.280000 01
  1  0.141996E 03    2  0.141996E 03    3  0.142011E 03    4  0.142011E 03    5  0.142130E 03    6  0.142130E 03
  7  0.142348E 03    8  0.142348E 03    9  0.142656E 03   10  0.142656E 03   11  0.143046E 03   12  0.143046E 03
 13  0.143511E 03   14  0.143511E 03   15  0.144046E 03   16  0.144046E 03   17  0.144647E 03   18  0.144647E 03
 19  0.145310E 03   20  0.145310E 03   21  0.146029E 03   22  0.146029E 03   23  0.146802E 03   24  0.146802E 03
 25  0.147622E 03   26  0.147622E 03   27  0.148485E 03   28  0.148485E 03   29  0.149388E 03   30  0.149388E 03
 31  0.150310E 03   32  0.150310E 03   33  0.150783E 03   34  0.150783E 03   35  0.151037E 03   36  0.151037E 03
 37  0.151232E 03   38  0.151232E 03   39  0.151231E 03   40  0.151231E 03

TIME= 0.600000 01
  1  0.150741E 03    2  0.150741E 03    3  0.150749E 03    4  0.150749E 03    5  0.150813E 03    6  0.150813E 03
  7  0.150930E 03    8  0.150930E 03    9  0.151094E 03   10  0.151094E 03   11  0.151301E 03   12  0.151301E 03
 13  0.151548E 03   14  0.151548E 03   15  0.151831E 03   16  0.151831E 03   17  0.152147E 03   18  0.152147E 03
 19  0.152495E 03   20  0.152495E 03   21  0.152870E 03   22  0.152870E 03   23  0.153272E 03   24  0.153272E 03
 25  0.153696E 03   26  0.153696E 03   27  0.154140E 03   28  0.154140E 03   29  0.154605E 03   30  0.154605E 03
 31  0.155074E 03   32  0.155074E 03   33  0.155323E 03   34  0.155323E 03   35  0.155463E 03   36  0.155463E 03
 37  0.155546E 03   38  0.155546E 03   39  0.155544E 03   40  0.155544E 03

TIME= 0.266800 03
  1  0.152120E 03    2  0.152120E 03    3  0.152106E 03    4  0.152106E 03    5  0.151995E 03    6  0.151995E 03
  7  0.151790E 03    8  0.151790E 03    9  0.151501E 03   10  0.151501E 03   11  0.151136E 03   12  0.151136E 03
 13  0.150697E 03   14  0.150697E 03   15  0.150196E 03   16  0.150196E 03   17  0.149631E 03   18  0.149631E 03
 19  0.149006E 03   20  0.149006E 03   21  0.148332E 03   22  0.148332E 03   23  0.147605E 03   24  0.147605E 03
 25  0.146836E 03   26  0.146836E 03   27  0.146024E 03   28  0.146024E 03   29  0.145180E 03   30  0.145180E 03
 31  0.144308E 03   32  0.144308E 03   33  0.143939E 03   34  0.143939E 03   35  0.143724E 03   36  0.143725E 03
 37  0.143420E 03   38  0.143420E 03   39  0.143417E 03   40  0.143417E 03

TIME= 0.270000 03
  1  0.143879E 01    2  0.143879E 03    3  0.143871E 03    4  0.143871E 03    5  0.143811E 03    6  0.143811E 03
  7  0.143700E 03    8  0.143700E 03    9  0.143544E 03   10  0.143544E 03   11  0.143347E 03   12  0.143347E 03
 13  0.143113E 03   14  0.143113E 03   15  0.142844E 03   16  0.142844E 03   17  0.142543E 03   18  0.142543E 03
 19  0.142213E 03   20  0.142213E 03   21  0.141855E 03   22  0.141855E 03   23  0.141473E 03   24  0.141473E 03
 25  0.141071E 03   26  0.141071E 03   27  0.140647E 03   28  0.140647E 03   29  0.140210E 03   30  0.140210E 03
 31  0.139763E 03   32  0.139763E 03   33  0.139503E 03   34  0.139503E 03   35  0.139494E 03   36  0.139494E 03
 37  0.139298E 03   38  0.139298E 03   39  0.139296E 03   40  0.139296E 03

TIME= 0.276000 03
  1  0.137544E 03    2  0.137544E 03    3  0.137541E 03    4  0.137541E 03    5  0.137524E 03    6  0.137524E 03
  7  0.137492E 03    8  0.137492E 03    9  0.137447E 03   10  0.137447E 03   11  0.137391E 03   12  0.137391E 03
 13  0.137324E 03   14  0.137324E 03   15  0.137247E 03   16  0.137247E 03   17  0.137161E 03   18  0.137161E 03
 19  0.137066E 03   20  0.137066E 03   21  0.136965E 03   22  0.136965E 03   23  0.136854E 03   24  0.136854E 03
 25  0.136740E 03   26  0.136740E 03   27  0.136618E 03   28  0.136618E 03   29  0.136492E 03   30  0.136493E 03
 31  0.136360E 03   32  0.136360E 03   33  0.136267E 03   34  0.136267E 03   35  0.136338E 03   36  0.136338E 03
 37  0.136222E 03   38  0.136222E 03   39  0.136221E 03   40  0.136221E 03

TIME= 0.288000 03
  1  0.135196E 03    2  0.135196E 03    3  0.135195E 03    4  0.135195E 03    5  0.135194E 03    6  0.135194E 03
  7  0.135192E 03    8  0.135192E 03    9  0.135189E 03   10  0.135189E 03   11  0.135184E 03   12  0.135184E 03
 13  0.135179E 03   14  0.135179E 03   15  0.135173E 03   16  0.135173E 03   17  0.135166E 03   18  0.135166E 03
 19  0.135159E 03   20  0.135159E 03   21  0.135152E 03   22  0.135151E 03   23  0.135142E 03   24  0.135142E 03
 25  0.135134E 03   26  0.135134E 03   27  0.135123E 03   28  0.135124E 03   29  0.135114E 03   30  0.135115E 03
 31  0.135110E 03   32  0.135110E 03   33  0.135068E 03   34  0.135067E 03   35  0.135107E 03   36  0.135108E 03
 37  0.135082E 03   38  0.135082E 03   39  0.135081E 03   40  0.135081E 03
```

Table H-6 Cont'd

Table 5.4 (cont.)

```
MECHANICAL PROPERTIES INFORMATION

VISCOELASTIC PARAMETER STUDY B/A=5.00 B=4 EQUIVALENT-PLANE STRAIN

     NUMBER OF MATERIALS................................     2
     NUMBER OF ELEMENTS WITH REDEFINED MATERIALS.......     0
     NUMBER OF NODES WITH SPECIFIED BOUNDARY VALUES....    40
     NUMBER OF PRESSURE CARDS..........................     0
     NUMBER OF ELEMENTS FOR DAMAGE EVALUATION..........     2
     ANGULAR VELOCITY AMPLITUDE..........          0.0
     ANGULAR VELOCITY FUNCTION...........          0
     AXIAL ACCELERATION AMPLITUDE........          0.0
     AXIAL ACCELERATION FUNCTION.........          0
     DISPLACEMENT PRINT CONTROL FLAG.....          0


MATERIALS DATA
 MATERIAL     TERMS IN       POINTS IN   EQUILIBRIUM         BULK      EXPANSION           WEIGHT
  NUMBER   PRONY SERIES  SHIFT FUNCTION      MODULUS      MODULUS    COEFFICIENT          DENSITY

       1             16              12       33.300   500000.000   0.56700E-04    0.0
       2              0               0  11500000.000 25000000.000  0.59000E-05    0.0


SERIES REPRESENTATION OF THE SHEAR RELAXATION FUNCTIONS
 MATERIAL          TERM IN      COEFFICIENT         EXPONENT
  NUMBER         THE SERIES       ALPHA(I)           BETA(I)

       1                 1    0.86120E 04      0.30000E 11
       1                 2    0.40290E 04      0.30000E 10
       1                 3    0.23064E 04      0.30000E 09
       1                 4    0.13052E 04      0.30000E 08
       1                 5    0.81140E 03      0.30000E 07
       1                 6    0.44730E 03      0.30000E 06
       1                 7    0.34463E 03      0.30000E 05
       1                 8    0.15266E 03      0.30000E 04
       1                 9    0.88600E 02      0.30000E 03
       1                10    0.45910E 02      0.30000E 02
       1                11    0.54600E-01      0.30000E 01
       1                12    0.37820E 02      0.30000E 00
       1                13   -0.96760E 00      0.30000E-01
       1                14    0.20083E 02      0.30000E-02
       1                15    0.14232E 01      0.30000E-03
       1                16    0.10471E 01      0.30000E-04

       2       ELASTIC MATERIAL (NO SERIES EXPANSION)


   TABULATION OF TIME-TEMPERATURE SHIFT FUNCTIONS
   MATERIAL     POINT      TEMPERATURE     SHIFT FACTOR
    NUMBER     NUMBER          T(I)       LOG-10 A(T(I))

         1          1      -100.000           8.2800
         1          2       -75.000           6.0000
         1          3       -50.000           4.6000
         1          4       -25.000           3.5400
         1          5         0.0             2.5200
         1          6        25.000           1.6000
         1          7        50.000           0.8000
         1          8        77.000           0.0
         1          9       100.000          -0.5600
         1         10       125.000          -1.2200
         1         11       150.000          -1.8200
         1         12       200.000          -2.9400

         2       TEMPERATURE INDEPENDENT MATERIAL
```

Table H-6 Cont'd.

Table 5.4 (cont.)

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NODAL POINT BOUNDARY CONDITIONS

                      R A D I A L                                A X I A L
 NODE      TYPE        BOUNDARY     FUNCTION      TYPE            BOUNDARY      FUNCTION
NUMBER     CODE           VALUE       NUMBER      CODE               VALUE        NUMBER

   1         0      0.0                  0          1        0.0                     0
   2         0      0.0                  0          1        0.0                     0
   3         0      0.0                  0          1        0.0                     0
   4         0      0.0                  0          1        0.0                     0
   5         0      0.0                  0          1        0.0                     0
   6         0      0.0                  0          1        0.0                     0
   7         0      0.0                  0          1        0.0                     0
   8         0      0.0                  0          1        0.0                     0
   9         0      0.0                  0          1        0.0                     0
  10         0      0.0                  0          1        0.0                     0
  11         0      0.0                  0          1        0.0                     0
  12         0      0.0                  0          1        0.0                     0
  13         0      0.0                  0          1        0.0                     0
  14         0      0.0                  0          1        0.0                     0
  15         0      0.0                  0          1        0.0                     0
  16         0      0.0                  0          1        0.0                     0
  17         0      0.0                  0          1        0.0                     0
  18         0      0.0                  0          1        0.0                     0
  19         0      0.0                  0          1        0.0                     0
  20         0      0.0                  0          1        0.0                     0
  21         0      0.0                  0          1        0.0                     0
  22         0      0.0                  0          1        0.0                     0
  23         0      0.0                  0          1        0.0                     0
  24         0      0.0                  0          1        0.0                     0
  25         0      0.0                  0          1        0.0                     0
  26         0      0.0                  0          1        0.0                     0
  27         0      0.0                  0          1        0.0                     0
  28         0      0.0                  0          1        0.0                     0
  29         0      0.0                  0          1        0.0                     0
  30         0      0.0                  0          1        0.0                     0
  31         0      0.0                  0          1        0.0                     0
  32         0      0.0                  0          1        0.0                     0
  33         0      0.0                  0          1        0.0                     0
  34         0      0.0                  0          1        0.0                     0
  35         0      0.0                  0          1        0.0                     0
  36         0      0.0                  0          1        0.0                     0
  37         0      0.0                  0          1        0.0                     0
  38         0      0.0                  0          1        0.0                     0
  39         0      0.0                  0          1        0.0                     0
  40         0      0.0                  0          1        0.0                     0

DAMAGE PARAMETERS ... NEG. EL. NO. = DAMAGE BASED ON MAX. PRINC. R-Z STRESS
                      POS. EL. NO. = DAMAGE BASED ON HOOP STRESS

ELEMENT SIGMA(T-ZERO)  T-ZERO  SIGMA(CRIT.)     B

     1        130.00     0.02           0.0   8.43
   -18        130.00     0.02           0.0   8.43
```

Table H-6 Cont'd

Table 5.4 (cont.)

```
                                        TIME = 1.0000000000D-01

     SIGR      SIGTH       SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM
 1 -8.00E-03 -1.30E-01 -7.20E-02 -2.16E-05  3.63E-04 -3.63E-04  0.0       -1.36E-10  1.35E 02
            DAMAGE RATE = 0.0          DAMAGE = 4.239E-29
 2 -2.00E-02 -1.18E-01 -7.21E-02 -5.62E-08  2.62E-04 -2.62E-04  0.0       -3.62E-10  1.35E 02
 3 -4.05E-02 -1.04E-01 -7.21E-02 -8.00E-08  1.80E-04 -1.80E-04  0.0       -3.62E-10  1.35E 02
 4 -4.91E-02 -9.51E-02 -7.21E-02 -6.01E-08  1.31E-04 -1.31E-04  0.0       -3.29E-10  1.35E 02
 5 -5.46E-02 -8.95E-02 -7.21E-02 -8.01E-08  9.94E-05 -9.95E-05  0.0       -4.61E-10  1.35E 02
 6 -5.83E-02 -8.58E-02 -7.21E-02 -8.91E-08  7.61E-05 -7.62E-05  0.0       -5.39E-10  1.35E 02
 7 -6.10E-02 -8.32E-02 -7.20E-02 -9.53E-08  6.30E-05 -6.31E-05  0.0       -5.54E-10  1.35E 02
 8 -6.29E-02 -8.11E-02 -7.20E-02 -7.01E-08  5.19E-05 -5.20E-05  0.0       -3.98E-10  1.35E 02
 9 -6.44E-02 -7.97E-02 -7.21E-02 -3.88E-08  4.36E-05 -4.36E-05  0.0       -2.19E-10  1.35E 02
10 -6.55E-02 -7.87E-02 -7.22E-02 -6.89E-08  3.79E-05 -3.70E-05  0.0       -4.01E-10  1.35E 02
11 -6.65E-02 -7.84E-02 -7.28E-02 -7.17E-08  3.58E-05 -3.17E-05  0.0       -4.38E-10  1.35E 02
12 -6.73E-02 -7.95E-02 -7.48E-02 -4.43E-08  4.23E-05 -2.69E-05  0.0       -2.56E-10  1.35E 02
13 -6.82E-02 -8.43E-02 -8.05E-02 -5.49E-08  6.98E-05 -2.17E-05  0.0       -3.22E-10  1.35E 02
14 -6.95E-02 -9.55E-02 -9.30E-02 -4.58E-08  1.37E-04 -1.42E-05  0.0       -2.78E-10  1.36E 02
15 -7.10E-02 -1.10E-01 -1.10E-01 -8.10E-09  2.08E-04 -2.07E-06  0.0        7.68E-11  1.37E 02
16 -7.36E-02 -1.38E-01 -1.40E-01  9.38E-09  4.15E-04  1.06E-05  0.0        8.81E-11  1.38E 02
17 -7.50E-02 -1.53E-01 -1.56E-01 -3.75E-09  5.11E-04  1.89E-05  0.0       -1.42E-11  1.38E 02
18 -7.60E-02 -1.62E-01 -1.65E-01  2.42E-08  5.77E-04  2.56E-05  0.0        1.73E-10  1.39E 02
            DAMAGE RATE = 0.0          DAMAGE = 8.414E-32
19 -8.35E-02  7.61E 00 -8.89E 02  1.25E-02  2.67E-05  2.90E-05  0.0        5.40E-10  1.35E 02

                                        TIME = 3.2000000000D-01

     SIGR      SIGTH       SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM
 1 -3.75E-02 -6.30E-01 -3.34E-01 -7.25E-07  2.10E-03 -2.10E-03  0.0       -5.58E-09  1.35E 02
            DAMAGE RATE = 0.0          DAMAGE = 4.239E-29
 2 -1.21E-01 -5.49E-01 -3.35E-01 -9.33E-07  1.52E-03 -1.52E-03  0.0       -7.93E-09  1.35E 02
 3 -1.88E-01 -4.81E-01 -3.35E-01 -5.05E-07  1.04E-03 -1.04E-03  0.0       -4.26E-09  1.35E 02
 4 -2.28E-01 -4.42E-01 -3.35E-01 -1.93E-07  7.57E-04 -7.56E-04  0.0       -1.56E-09  1.35E 02
 5 -2.53E-01 -4.16E-01 -3.35E-01 -1.11E-07  5.78E-04 -5.75E-04  0.0       -1.14E-09  1.35E 02
 6 -2.71E-01 -3.99E-01 -3.36E-01 -2.15E-07  4.58E-04 -4.51E-04  0.0       -2.32E-09  1.35E 02
 7 -2.83E-01 -3.88E-01 -3.37E-01 -2.97E-07  3.77E-04 -3.63E-04  0.0       -2.71E-09  1.35E 02
 8 -2.93E-01 -3.81E-01 -3.39E-01 -2.28E-07  3.25E-04 -2.97E-04  0.0       -2.11E-09  1.35E 02
 9 -3.00E-01 -3.77E-01 -3.43E-01 -1.78E-07  2.97E-04 -2.40E-04  0.0       -1.61E-09  1.35E 02
10 -3.06E-01 -3.77E-01 -3.48E-01 -1.87E-07  2.94E-04 -2.04E-04  0.0       -1.75E-09  1.36E 02
11 -3.11E-01 -3.80E-01 -3.57E-01 -1.19E-07  3.18E-04 -1.67E-04  0.0       -1.30E-09  1.36E 02
12 -3.16E-01 -3.88E-01 -3.69E-01 -9.30E-08  3.74E-04 -1.32E-04  0.0       -9.69E-10  1.36E 02
13 -3.21E-01 -3.99E-01 -3.86E-01 -1.28E-07  4.67E-04 -9.73E-05  0.0       -1.25E-09  1.37E 02
14 -3.26E-01 -4.16E-01 -4.12E-01 -1.35E-07  6.31E-04 -5.94E-05  0.0       -1.36E-09  1.38E 02
15 -3.32E-01 -4.35E-01 -4.35E-01 -3.90E-08  7.78E-04 -1.78E-05  0.0       -2.83E-10  1.39E 02
16 -3.36E-01 -4.53E-01 -4.54E-01  4.56E-08  9.36E-04  1.60E-05  0.0        4.97E-10  1.41E 02
17 -3.38E-01 -4.62E-01 -4.66E-01  4.29E-08  1.03E-03  3.41E-05  0.0        3.15E-10  1.41E 02
18 -3.40E-01 -4.68E-01 -4.74E-01  3.23E-08  1.08E-03  4.70E-05  0.0        3.55E-10  1.42E 02
            DAMAGE RATE = 0.0          DAMAGE = 8.414E-32
19 -1.99E-01  2.72E 01 -1.20E 03  1.92E-02  5.23E-05  5.35E-05  0.0        8.37E-10  1.42E 02

                                        TIME = 1.0000000000D 00

     SIGR      SIGTH       SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM
 1 -1.11E-01 -1.86E 00 -9.93E-01 -1.21E-06  7.43E-03 -7.31E-03  0.0       -1.59E-08  1.36E 02
            DAMAGE RATE = 0.0          DAMAGE = 4.239E-29
 2 -3.57E-01 -1.62E 00 -9.97E-01 -2.22E-06  5.39E-03 -5.26E-03  0.0       -2.72E-08  1.36E 02
 3 -5.55E-01 -1.42E 00 -9.99E-01 -1.92E-06  3.73E-03 -3.58E-03  0.0       -7.23E-08  1.36E 02
 4 -6.73E-01 -1.31E 00 -1.00E 00 -1.59E-06  2.76E-03 -2.59E-03  0.0       -1.74E-08  1.36E 02
 5 -7.49E-01 -1.24E 00 -1.01E 00 -1.70E-06  2.16E-03 -1.94E-03  0.0       -1.75E-08  1.36E 02
 6 -8.02E-01 -1.19E 00 -1.01E 00 -1.95E-06  1.77E-03 -1.50E-03  0.0       -2.12E-08  1.37E 02
 7 -8.40E-01 -1.16E 00 -1.02E 00 -1.97E-06  1.52E-03 -1.18E-03  0.0       -2.19E-08  1.37E 02
 8 -8.69E-01 -1.14E 00 -1.03E 00 -1.45E-06  1.37E-03 -9.39E-04  0.0       -1.67E-08  1.38E 02
 9 -8.91E-01 -1.13E 00 -1.04E 00 -8.07E-07  1.28E-03 -7.45E-04  0.0       -1.01E-08  1.38E 02
10 -9.09E-01 -1.12E 00 -1.06E 00 -3.01E-07  1.24E-03 -5.71E-04  0.0       -4.26E-09  1.39E 02
11 -9.24E-01 -1.12E 00 -1.07E 00 -1.45E-07  1.25E-03 -4.58E-04  0.0       -2.31E-09  1.40E 02
12 -9.38E-01 -1.13E 00 -1.09E 00 -8.64E-08  1.30E-03 -3.42E-04  0.0       -1.61E-09  1.41E 02
13 -9.50E-01 -1.13E 00 -1.11E 00 -5.68E-09  1.37E-03 -2.36E-04  0.0       -1.39E-09  1.42E 02
14 -9.61E-01 -1.14E 00 -1.13E 00 -1.35E-08  1.47E-03 -1.37E-04  0.0       -9.91E-10  1.43E 02
15 -9.72E-01 -1.16E 00 -1.15E 00 -9.24E-08  1.60E-03 -4.35E-05  0.0        7.79E-10  1.44E 02
16 -9.79E-01 -1.17E 00 -1.17E 00  8.17E-08  1.71E-03  2.34E-05  0.0        1.11E-09  1.45E 02
17 -9.83E-01 -1.17E 00 -1.18E 00  2.10E-08  1.76E-03  5.59E-05  0.0        3.87E-10  1.46E 02
18 -9.85E-01 -1.18E 00 -1.18E 00  5.03E-08  1.80E-03  7.76E-05  0.0        7.53E-10  1.46E 02
            DAMAGE RATE = 0.0          DAMAGE = 8.414E-32
19 -5.35E-01  7.55E 01 -1.95E 03  3.76E-02  8.51E-05  8.84E-05  0.0        1.64E-09  1.46E 02
```

Table H-6 Cont'd

APPENDIX H

NOT REPRODUCIBLE

Table 5.4 (cont.)

```
                                         TIME = 2.30000000000 00

      SIGR        SIGTH        SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM

 1 -2.63E-01 -4.26E 00 -2.81E 00 -1.01E-06  1.90E-02 -1.73E-02   0.0      -2.53E-06  1.42E 02
              DAMAGE RATE = 0.0            DAMAGE = 4.237E-29
 2 -8.13E-01 -3.09E 00 -2.32E 00 -3.02E-05  1.39E-02 -1.27E-02   0.0      -4.41E-08  1.42E 02
 3 -1.27E 00 -3.24E 00 -2.32E 00 -2.92E-06  9.74E-03 -5.51E-03   0.0      -4.01E-08  1.42E 02
 4 -1.53E 00 -2.97E 00 -2.32E 00 -2.34E-06  7.29E-03 -5.02E-03   0.0      -3.19E-08  1.43E 02
 5 -1.[illegible]E 00 -2.80E 00 -2.33E 00 -2.55E-06  5.75E-03 -4.44E-03   0.0      -3.38E-08  1.43E 02
 6 -1.82E 00 -2.69E 00 -2.33E 00 -3.59E-06  4.73E-03 -3.33E-03   0.0      -4.27E-08  1.43E 02
 7 -1.91E 00 -2.62E 00 -2.34E 00 -1.48E-06  4.04E-03 -2.55E-03   0.0      -4.51E-08  1.44E 02
 8 -1.97E 00 -2.56E 00 -2.35E 00 -2.77E-06  3.55E-03 -1.75E-03   0.0      -3.50E-08  1.44E 02
 9 -2.02E 00 -2.52E 00 -2.36E 00 -1.97E-06  3.20E-03 -1.51E-03   0.0      -2.46E-08  1.45E 02
10 -2.06E 00 -2.50E 00 -2.37E 00 -1.17E-06  2.90E-03 -1.15E-03   0.0      -1.43E-08  1.46E 02
11 -2.09E 00 -2.48E 00 -2.38E 00 -4.81E-07  2.80E-03 -8.62E-04   0.0      -6.39E-09  1.46E 02
12 -2.11E 00 -2.46E 00 -2.40E 00 -2.29E-07  2.69E-03 -6.19E-04   0.0      -3.43E-09  1.47E 02
13 -2.14E 00 -2.45E 00 -2.41E 00  2.60E-09  2.63E-03 -4.12E-04   0.0      -9.36E-10  1.48E 02
14 -2.15E 00 -2.43E 00 -2.42E 00  7.40E-08  2.60E-03 -2.33E-04   0.0      -3.64E-11  1.49E 02
15 -2.17E 00 -2.44E 00 -2.44E 00  1.57E-07  2.60E-03 -7.40E-05   0.0       1.83E-09  1.50E 02
16 -2.18E 00 -2.44E 00 -2.45E 00  1.00E-07  2.61E-03  3.20E-05   0.0       1.63E-09  1.51E 02
17 -2.19E 00 -2.44E 00 -2.46E 00  1.49E-08  2.62E-03  8.19E-05   0.0       4.00E-10  1.51E 02
18 -2.19E 00 -2.44E 00 -2.48E 00  9.47E-08  2.63E-03  1.14E-04   0.0       1.44E-09  1.51E 02
              DAMAGE RATE = 0.0            DAMAGE = 8.414E-32
19 -1.10E 00  1.01E 02 -2.82E 03  6.27E-02  1.22E-01  1.29E-04   0.0       2.73E-09  1.51E 02

                                         TIME = 2.40000000000 01

      SIGR        SIGTH        SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM

 1 -3.97E-01 -6.54E 00 -3.62E 00 -3.70E-06  4.44E-02 -4.00E-02   0.0      -1.05E-07  1.60E 02
              DAMAGE RATE = 0.0            DAMAGE = 4.231E-29
 2 -1.20E 00 -5.73E 00 -3.63E 00 -4.92E-06  3.30E-02 -2.88E-02   0.0      -1.55E-07  1.60E 02
 3 -1.56E 00 -5.00E 00 -3.63E 00 -5.02E-06  2.33E-02 -1.90E-02   0.0      -1.[illegible]E-07  1.60E 02
 4 -2.37E 00 -4.50E 00 -3.63E 00 -4.79E-06  1.75E-02 -1.33E-02   0.0      -1.37E-07  1.60E 02
 5 -2.63E 00 -4.31E 00 -3.63E 00 -5.24E-06  1.38E-02 -9.50E-03   0.0      -1.48E-07  1.60E 02
 6 -2.81E 00 -4.13E 00 -3.63E 00 -5.95E-06  1.13E-02 -7.07E-03   0.0      -1.74E-07  1.60E 02
 7 -2.94E 00 -4.01E 00 -3.63E 00 -5.81E-06  9.54E-03 -5.30E-03   0.0      -1.73E-07  1.60E 02
 8 -3.03E 00 -3.91E 00 -3.63E 00 -4.79E-06  8.23E-03 -3.94E-03   0.0      -1.43E-07  1.60E 02
 9 -3.10E 00 -3.84E 00 -3.62E 00 -3.04E-06  7.24E-03 -3.00E-03   0.0      -1.04E-07  1.60E 02
10 -3.16E 00 -3.79E 00 -3.62E 00 -2.21E-06  6.47E-03 -2.23E-03   0.0      -6.21E-08  1.60E 02
11 -3.20E 00 -3.74E 00 -3.62E 00 -1.72E-06  5.87E-03 -1.62E-03   0.0      -2.86E-08  1.60E 02
12 -3.24E 00 -3.71E 00 -3.62E 00 -3.87E-07  5.38E-03 -1.14E-03   0.0      -1.15E-08  1.60E 02
13 -3.27E 00 -3.68E 00 -3.62E 00 -1.04E-07  4.99E-03 -7.37E-04   0.0      -4.06E-09  1.60E 02
14 -3.29E 00 -3.65E 00 -3.62E 00  9.24E-08  4.66E-03 -4.05E-04   0.0       1.15E-09  1.60E 02
15 -3.31E 00 -3.63E 00 -3.62E 00  1.93E-07  4.38E-03 -1.31E-04   0.0       5.56E-09  1.60E 02
16 -3.32E 00 -3.62E 00 -3.62E 00  1.49E-07  4.20E-03  4.62E-05   0.0       5.14E-09  1.60E 02
17 -3.33E 00 -3.61E 00 -3.62E 00  6.87E-08  4.13E-03  1.25E-04   0.0       2.61E-09  1.60E 02
18 -3.33E 00 -3.61E 00 -3.62E 00  7.54E-08  4.08E-03  1.73E-04   0.0       3.20E-09  1.60E 02
              DAMAGE RATE = 0.0            DAMAGE = 8.414E-32
19 -1.77E 00  2.40E 02 -4.34E 03  1.05E-01  1.89E-06  2.30E-04   0.0       4.54E-09  1.60E 02

                                         TIME = 4.80000000000 01

      SIGR        SIGTH        SIGZ       SIGRZ        EPR        EPTH       EPZ        EPRZ       TEM

 1  3.52E-02  6.12E-01  3.40E-01 -5.58E-07  3.15E-04 -2.01E-04   0.0      -5.64E-08  1.35E 02
              DAMAGE RATE = 4.207E-17      DAMAGE = 4.782E-15
 2  1.14E-01  5.32E-01  3.43E-01 -1.11E-07  2.33E-04 -1.93E-04   0.0      -7.23E-08  1.35E 02
 3  1.81E-01  4.67E-01  3.40E-01 -3.07E-07  1.05E-04 -1.30E-04   0.0      -7.59E-08  1.35E 02
 4  2.20E-01  4.24E-01  3.43E-01 -4.75E-07  1.24E-04 -8.97E-05   0.0      -7.15E-08  1.35E 02
 5  2.45E-01  4.05E-01  3.41E-01 -3.02E-08  9.81E-05 -6.41E-05   0.0      -6.52E-08  1.35E 02
 6  2.52E-01  3.86E-01  3.41E-01  8.31E-07  8.03E-05 -4.60E-05   0.0      -7.46E-08  1.35E 02
 7  2.74E-01  3.70E-01  3.41E-01 -1.25E-07  6.57E-05 -3.46E-05   0.0      -7.66E-08  1.35E 02
 8  2.83E-01  3.60E-01  3.41E-01 -7.39E-07  5.57E-05 -2.37E-05   0.0      -7.44E-08  1.35E 02
 9  2.90E-01  3.51E-01  3.41E-01 -8.74E-07  4.84E-05 -1.91E-05   0.0      -5.84E-08  1.35E 02
10  2.96E-01  3.50E-01  3.41E-01 -2.69E-07  4.25E-05 -1.40E-05   0.0      -2.92E-08  1.35E 02
11  3.00E-01  3.52E-01  3.41E-01  1.63E-08  3.71E-05 -1.01E-05   0.0      -1.17E-08  1.35E 02
12  3.03E-01  3.40E-01  3.41E-01 -1.12E-08  3.25E-05 -7.12E-06   0.0      -5.88E-09  1.35E 02
13  3.06E-01  3.45E-01  3.41E-01  6.55E-08  2.85E-05 -4.74E-06   0.0      -9.81E-10  1.35E 02
14  3.09E-01  3.43E-01  3.40E-01  1.14E-07  2.48E-05 -2.87E-06   0.0       1.00E-09  1.35E 02
15  3.10E-01  3.41E-01  3.40E-01  1.31E-07  2.21E-05 -1.40E-06   0.0       4.45E-09  1.35E 02
16  3.11E-01  3.39E-01  3.40E-01  6.93E-08  1.92E-05 -4.97E-07   0.0       3.49E-09  1.35E 02
17  3.12E-01  3.39E-01  3.42E-01  2.49E-09  1.33E-05 -1.40E-07   0.0       1.10E-09  1.35E 02
18  3.12E-01  3.38E-01  3.37E-01 -4.02E-09  1.70E-05  7.14E-08   0.0       3.79E-10  1.35E 02
              DAMAGE RATE = 2.910E-19      DAMAGE = 3.774E-17
19  1.05E-01 -2.09E 01 -7.51E 01 -2.15E-03  1.19E-06  1.84E-07   0.0      -9.37E-11  1.35E 02
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Table H-6 Cont'd

APPENDIX H

NOT REPRODUCIBLE

Table 5.4 (cont.)

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                                     TIME = 7.2000000000 01

      SIGR        SIGTH        SIGZ       SIGRZ        EPR         EPTH        EPZ        EPRZ       TEM

  1  5.046E-01  9.617E 00  5.030E 00  1.949E-05 -4.447E-02  4.056E-02  0.0        -4.308E-09  1.10E 02
               DAMAGE RATE = 7.551E-06   DAMAGE = 1.847E-06
  2  1.765E 00  7.990E 00  5.094E 00  4.910E-06 -3.249E-02  2.874E-02  0.0         2.285E-08  1.10E 02
  3  2.747E 00  7.003E 00  5.094E 00  5.780E-06 -2.126E-02  1.490E-02  0.0         7.856E-08  1.10E 02
  4  3.532E 00  6.442E 00  5.094E 00  4.416E-06 -1.753E-02  1.526E-02  0.0         1.766E-08  1.10E 02
  5  3.695E 00  6.052E 00  5.072E 00  5.502E-06 -1.439E-02  [illegible]  0.0       1.448E-08  1.10E 02
  6  3.994E 00  5.802E 00  5.094E 00  6.708E-06 -1.113E-02  7.600E-03  0.0         4.606E-08  1.10E 02
  7  4.125E 00  5.622E 00  5.094E 00  6.075E-05 -9.325E-03  5.228E-03  0.0         3.855E-08  1.10E 02
  8  4.25[illegible] 00  5.447E 00  5.094E 00  4.675E-05 -8.225E-03  3.790E-03  0.0    2.338E-08  1.10E 02
  9  4.35[illegible] 00  5.395E 00  5.094E 00  3.235E-05 -7.235E-03  2.997E-03  0.0    1.362E-08  1.10E 02
 10  4.43[illegible] 00  5.318E 00  5.094E 00  2.031E-05 -6.405E-03  2.222E-03  0.0    1.278E-08  1.10E 02
 11  4.495E 00  5.25[illegible] 00  5.092E 00  1.094E-06 -5.560E-03  1.626E-03  0.0    8.500E-09  1.10E 02
 12  4.54[illegible] 00  5.202E 00  5.094E 00  2.416E-07 -5.037E-03  1.313E-03  0.0   -1.213E-09  1.10E 02
 13  4.58[illegible] 00  5.160E 00  5.094E 00  4.549E-06 -4.743E-03  7.338E-04  0.0   -3.530E-10  1.10E 02
 14  4.622E 00  5.133E 00  5.092E 00  1.79[illegible]-03 -4.555E-03  4.046E-04  0.0    1.623E-09  1.10E 02
 15  4.646E 00  5.102E 00  5.094E 00 -4.741E-08 -4.377E-03  1.228E-04  0.0         1.448E-09  1.10E 02
 16  4.660E 00  5.086E 00  5.092E 00 -1.218E-07 -4.206E-03 -4.922E-05  0.0        -4.215E-10  1.10E 02
 17  4.675E 00  5.082E 00  5.092E 00 -1.506E-07 -4.128E-03 -1.262E-04  0.0        -2.275E-09  1.10E 02
 18  4.677E 00  5.072E 00  5.094E 00 -1.477E-07 -4.075E-03 -1.788E-04  0.0        -2.415E-09  1.10E 02
               DAMAGE RATE = 5.463E-10   DAMAGE = 9.747E-09
 19  2.445E 00 -3.394E 02  4.320E 03 -7.349E-02 -1.086E-04 -2.026E-04  0.0        -3.195E-09  1.10E 02

                                     TIME = 9.6000000000 01

      SIGR        SIGTH        SIGZ       SIGRZ        EPR         EPTH        EPZ        EPRZ       TEM

  1  1.111E 00  1.680E 01  1.030E 01  7.19[illegible]-06 -8.988E-02  9.113E-02  0.0     9.72E-08  6.45E 01
               DAMAGE RATE = 7.223E-06   DAMAGE = 1.431E-04
  2  3.588E 00  1.602E 01  1.034E 01  1.13[illegible]-05 -6.615E-02  5.702E-02  0.0     1.608E-07  6.45E 01
  3  5.570E 00  1.426E 01  1.033E 01  1.211E-05 -4.606E-02  3.618E-02  0.0         1.066E-07  6.45E 01
  4  6.748E 00  1.318E 01  1.033E 01  1.00[illegible]-05 -3.651E-02  2.605E-02  0.0     1.40E-07   6.45E 01
  5  7.50[illegible] 00  1.233E 01  1.033E 01  1.17[illegible]-05 -2.775E-02  1.926E-02  0.0   1.60E-07  6.45E 01
  6  8.01[illegible] 00  1.183E 01  1.033E 01  1.35[illegible]-05 -2.427E-02  1.442E-02  0.0   1.40E-07  6.45E 01
  7  8.39[illegible] 00  1.149E 01  1.033E 01  1.320E-05 -1.910E-02  1.306E-02  0.0         1.87E-07   6.45E 01
  8  8.64[illegible] 00  1.12[illegible] 01  1.03[illegible] 01  1.09[illegible]-05 -1.605E-02  7.99E-03  0.0   1.54E-07  6.45E 01
  9  8.85E 00  1.105E 01  1.033E 01  7.62E-06 -1.455E-02  6.00E-03  0.0          1.10E-07   6.45E 01
 10  9.01E 00  1.095E 01  1.033E 01  4.91E-06 -1.306E-02  [illegible]  0.0          7.32E-08   6.45E 01
 11  9.13E 00  1.07[illegible] 01  1.035E 01  2.51E-06 -1.181E-02  3.24E-03  0.0    3.43E-08   6.45E 01
 12  9.23E 00  1.06[illegible] 01  1.03[illegible] 01  [illegible]-07 -1.05[illegible]-02  [illegible]  0.0   -4.67E-09  6.45E 01
 13  9.31E 00  1.03[illegible] 01  1.03[illegible] 01  3.57E-05 [illegible]  1.47E-03  0.0   -1.07E-10  6.45E 01
 14  9.35E 00  1.04[illegible] 01  1.03[illegible] 01 -[illegible]-08 -9.32E-03  8.09E-04  0.0   -8.24E-12  6.45E 01
 15  9.44E 00  1.04[illegible] 01  1.03[illegible] 01 -3.42E-07 -8.77E-03  2.65E-04  0.0   -4.61E-09  6.45E 01
 16  9.45E 00  1.03[illegible] 01  1.03[illegible] 01 -5.63E-07 -8.42E-03 -[illegible]-05  0.0  -1.02E-08  6.45E 01
 17  9.49E 00  1.03[illegible] 01  1.03[illegible] 01 -3.04E-07 -8.23E-03 -2.57E-04  0.0   -9.40E-09  6.45E 01
 18  9.50E 00  1.03[illegible] 01  1.03[illegible] 01 -3.00E-07 -8.15E-03 -3.57E-04  0.0   -6.[illegible]-09  6.45E 01
               DAMAGE RATE = 5.027E-06   DAMAGE = 9.347E-07
 19  4.44E 00 -6.77E 02  8.00E 03 -1.49E-01 -3.70E-04 -4.00E-04  0.0       -6.45E-09  6.45E 01

                                     TIME = 1.2000000000 02

      SIGR        SIGTH        SIGZ       SIGRZ        EPR         EPTH        EPZ        EPRZ       TEM

  1  1.75E 00  2.93E 01  1.02E 01  1.29E-05 -1.135E-01  1.22E-01  0.0       1.88E-07  5.57E 01
               DAMAGE RATE = 6.573E-05   DAMAGE = 1.579E-05
  2  5.64E 00  2.50E 01  1.03E 01  1.93E-05 -9.92E-02  9.04E-02  0.0       2.98E-07  5.57E 01
  3  8.77E 00  2.24E 01  1.03E 01  2.05E-05 -6.49E-02  5.71E-02  0.0       3.05E-07  5.57E 01
  4  1.00E 01  2.00E 01  1.03E 01  1.88E-05 -5.26E-02  3.98E-02  0.0       2.08E-07  5.57E 01
  5  1.18E 01  1.9[illegible] 01  1.03E 01  1.7[illegible]-05 -4.15E-02  2.69E-02  0.0   2.27E-07  5.57E 01
  6  1.26E 01  1.85E 01  1.03E 01  2.2[illegible]-05 -3.40E-02  2.12E-02  0.0   3.27E-07  5.57E 01
  7  1.32E 01  1.80E 01  1.03E 01  2.15E-05 -2.47E-02  1.59E-02  0.0       3.20E-07  5.57E 01
  8  1.36E 01  1.76E 01  1.03E 01  1.77E-05 -2.47E-02  1.23E-02  0.0       2.65E-07  5.57E 01
  9  1.39E 01  1.73E 01  1.03E 01  1.2[illegible]-05 -2.19E-02  [illegible]-03  0.0   1.42E-07  5.57E 01
 10  1.42E 01  1.72E 01  1.03E 01  7.4[illegible]-06 -1.95E-02  [illegible]-03  0.0   1.19E-07  5.57E 01
 11  1.44E 01  1.70E 01  1.03E 01  3.3[illegible]-06 -1.76E-02  [illegible]-03  0.0   5.30E-08  5.57E 01
 12  1.45E 01  1.07E 01  1.03E 01  [illegible]-07 -1.[illegible]-02  [illegible]-03  0.0   [illegible]-09  5.57E 01
 13  1.47E 01  1.0[illegible] 01  1.03E 01 -5.1[illegible]-08 -1.5[illegible]-02  [illegible]-03  0.0  -6.2[illegible]-10  5.57E 01
 14  1.46E 01  1.04E 01  1.03E 01 -1.4[illegible]-07 -1.4[illegible]-02  [illegible]-03  0.0  -3.4[illegible]-10  5.57E 01
 15  1.4[illegible] 01  1.03E 01  1.03E 01 -4.37E-07 -1.32E-02  [illegible]-04  0.0  -6.[illegible]-09  5.57E 01
 16  1.4[illegible] 01  1.0[illegible] 01  1.0[illegible] 01 -7.[illegible]-07 -1.2[illegible]-02 -1.50E-04  0.0  -1.47E-08  5.57E 01
 17  1.4[illegible] 01  1.0[illegible] 01  1.0[illegible] 01 -7.73E-07 -1.24E-02 -3.67E-04  0.0  -1.20E-08  5.57E 01
 18  1.4[illegible] 01  1.0[illegible] 01  1.0[illegible] 01 -4.[illegible]-07 -1.2[illegible]-02 -[illegible]-04  0.0  -1.07E-08  5.57E 01
               DAMAGE RATE = 4.5[illegible]-07   DAMAGE = 1.175E-05
 19  7.7[illegible] 03 -1.[illegible] 03  [illegible] 0[illegible] -[illegible]-01 -[illegible]-04 -[illegible]-04  0.0  -1.0[illegible]-08  5.57E 01
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Table H-6 Cont'd

NOT REPRODUCIBLE

Table 5.4 (cont.)

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                                  TIME = 1.44000000000 02

     SIGR        SIGTH       SIGZ       SIGRZ        EPR       EPTH        EPZ        EPRZ       TEM
 1   2.00E 00   4.35E 01   4.41E 01   1.90E-05  -1.30E-01   1.03E-01    0.0       2.77E-07   3.46E 01
             DAMAGE RATE = 5.307E-04   DAMAGE = 4.272E-03
 2   8.36E 00   3.77E 01   4.42E 01   2.43E-05  -1.32E-01   1.13E-01    0.0       4.27E-07   3.46E 01
 3   1.33E 01   3.33E 01   4.42E 01   3.01E-05  -9.32E-02   7.60E-02    0.0       4.25E-07   3.46E 01
 4   1.57E 01   3.05E 01   4.42E 01   2.71E-05  -7.31E-02   5.31E-02    0.0       3.73E-07   3.46E 01
 5   1.75E 01   2.87E 01   4.42E 01   2.04E-05  -5.34E-02   3.93E-02    0.0       4.01E-07   3.46E 01
 6   1.87E 01   2.75E 01   4.42E 01   3.35E-05  -4.33E-02   2.93E-02    0.0       4.57E-07   3.46E 01
 7   1.90E 01   2.67E 01   4.42E 01   3.15E-05  -3.82E-02   2.12E-02    0.0       4.42E-07   3.46E 01
 8   2.02E 01   2.61E 01   4.42E 01   2.46E-05  -3.30E-02   1.59E-02    0.0       3.59E-07   3.46E 01
 9   2.07E 01   2.56E 01   4.42E 01   1.84E-05  -2.70E-02   1.20E-02    0.0       2.00E-07   3.46E 01
10   2.10E 01   2.52E 01   4.42E 01   1.13E-05  -2.30E-02   5.71E-03    0.0       1.03E-07   3.46E 01
11   2.13E 01   2.49E 01   4.42E 01   4.72E-06  -2.30E-02   4.60E-03    0.0       6.86E-08   3.46E 01
12   2.16E 01   2.47E 01   4.42E 01   9.20E-07  -2.15E-02   4.39E-03    0.0       1.17E-08   3.46E 01
13   2.18E 01   2.45E 01   4.42E 01   2.33E-07  -2.00E-02   2.73E-03    0.0       4.44E-09   3.46E 01
14   2.19E 01   2.44E 01   4.42E 01   3.09E-07  -1.90E-02   1.01E-03    0.0       6.09E-09   3.46E 01
15   2.20E 01   2.44E 01   4.42E 01  -4.60E-09  -1.75E-02   5.05E-04    0.0      -2.14E-09   3.46E 01
16   2.21E 01   2.41E 01   4.42E 01  -6.25E-07  -1.69E-02  -2.30E-04    0.0      -1.31E-08   3.46E 01
17   2.22E 01   2.41E 01   4.42E 01  -7.24E-07  -1.65E-02  -5.22E-04    0.0      -1.20E-08   3.46E 01
18   2.22E 01   2.41E 01   4.42E 01  -4.17E-07  -1.63E-02  -7.22E-04    0.0      -1.13E-08   3.46E 01
             DAMAGE RATE = 2.079E-06   DAMAGE = 3.077E-05
19   1.15E 01  -1.57E 03   6.73E 04  -3.14E-01  -7.50E-04  -3.19E-04    0.0      -1.30E-08   3.45E 01

                                  TIME = 1.56000000000 02

     SIGR        SIGTH       SIGZ       SIGRZ        EPR       EPTH        EPZ        EPRZ       TEM
 1   3.56E 00   5.90E 01   3.31E 01   3.23E-05  -2.24E-01   2.03E-01    0.0       4.66E-07   5.41E 00
             DAMAGE RATE = 5.770E-04   DAMAGE = 2.547E-02
 2   1.15E 01   5.21E 01   3.32E 01   4.35E-05  -1.65E-01   1.44E-01    0.0       6.58E-07   5.41E 00
 3   1.79E 01   4.57E 01   3.32E 01   4.39E-05  -1.16E-01   9.51E-02    0.0       6.27E-07   5.40E 00
 4   2.16E 01   4.19E 01   3.32E 01   3.97E-05  -8.75E-02   5.63E-02    0.0       5.47E-07   5.40E 00
 5   2.41E 01   3.93E 01   3.32E 01   4.20E-05  -6.92E-02   4.79E-02    0.0       5.62E-07   5.40E 00
 6   2.57E 01   3.78E 01   3.32E 01   4.04E-05  -5.67E-02   3.34E-02    0.0       6.61E-07   5.40E 00
 7   2.69E 01   3.67E 01   3.32E 01   4.42E-05  -4.79E-02   2.69E-02    0.0       6.34E-07   5.40E 00
 8   2.77E 01   3.56E 01   3.32E 01   3.49E-05  -4.12E-02   1.97E-02    0.0       5.05E-07   5.40E 00
 9   2.84E 01   3.52E 01   3.32E 01   2.34E-05  -3.60E-02   1.50E-02    0.0       3.20E-07   5.40E 00
10   2.89E 01   3.47E 01   3.32E 01   1.47E-05  -3.24E-02   1.11E-02    0.0       9.50E-08   5.40E 00
11   2.93E 01   3.43E 01   3.32E 01   1.52E-05  -2.94E-02   8.09E-03    0.0      -6.38E-08   5.40E 00
12   2.96E 01   3.39E 01   3.32E 01   5.92E-07  -2.67E-02   5.03E-03    0.0      -1.15E-07   5.40E 00
13   2.99E 01   3.37E 01   3.32E 01  -1.80E-06  -2.50E-02   3.85E-03    0.0      -4.05E-08   5.40E 00
14   3.01E 01   3.35E 01   3.32E 01  -6.44E-07  -2.31E-02   2.00E-03    0.0      -1.95E-09   5.40E 00
15   3.03E 01   3.33E 01   3.32E 01  -2.13E-07  -2.17E-02   6.24E-04    0.0       1.00E-09   5.40E 00
16   3.04E 01   3.32E 01   3.32E 01  -5.12E-07  -2.10E-02  -2.04E-04    0.0      -9.05E-09   5.40E 00
17   3.05E 01   3.31E 01   3.32E 01  -6.29E-07  -2.05E-02  -6.50E-04    0.0      -1.31E-08   5.37E 00
18   3.05E 01   3.31E 01   3.32E 01  -3.54E-07  -2.04E-02  -9.08E-04    0.0      -1.23E-08   5.37E 00
             DAMAGE RATE = 4.030E-06   DAMAGE = 1.031E-04
19   1.57E 01  -2.14E 03   2.15E 04  -4.13E-01  -5.35E-04  -1.03E-03    0.0      -1.82E-08   5.41E 00

                                  TIME = 1.92000000000 02

     SIGR        SIGTH       SIGZ       SIGRZ        EPR       EPTH        EPZ        EPRZ       TEM
 1   4.67E 00   8.12E 01   4.56E 01   5.07E-05  -2.09E-01   2.43E-01    0.0       8.40E-07  -1.56E 01
             DAMAGE RATE = 7.345E-04   DAMAGE = 5.152E-02
 2   1.56E 01   7.00E 01   4.52E 01   6.13E-05  -1.78E-01   1.72E-01    0.0       1.02E-06  -1.56E 01
 3   2.43E 01   6.21E 01   4.51E 01   6.24E-05  -1.40E-01   1.19E-01    0.0       9.62E-07  -1.56E 01
 4   2.94E 01   5.79E 01   4.51E 01   5.79E-05  -1.05E-01   7.93E-02    0.0       8.72E-07  -1.56E 01
 5   3.27E 01   5.37E 01   4.51E 01   6.13E-05  -8.20E-02   5.74E-02    0.0       9.32E-07  -1.56E 01
 6   3.50E 01   5.14E 01   4.51E 01   6.07E-05  -6.79E-02   4.24E-02    0.0       1.04E-06  -1.56E 01
 7   3.65E 01   4.99E 01   4.51E 01   6.41E-05  -5.73E-02   3.17E-02    0.0       1.01E-06  -1.56E 01
 8   3.77E 01   4.87E 01   4.51E 01   5.24E-05  -4.94E-02   2.39E-02    0.0       8.34E-07  -1.56E 01
 9   3.86E 01   4.75E 01   4.51E 01   3.69E-05  -4.35E-02   1.79E-02    0.0       5.72E-07  -1.56E 01
10   3.93E 01   4.71E 01   4.51E 01   2.47E-05  -3.89E-02   1.33E-02    0.0       2.05E-07  -1.56E 01
11   3.98E 01   4.66E 01   4.51E 01   2.15E-05  -3.52E-02   9.00E-03    0.0       5.51E-09  -1.56E 01
12   4.03E 01   4.61E 01   4.51E 01   3.41E-06  -3.23E-02   6.70E-03    0.0      -7.34E-08  -1.57E 01
13   4.06E 01   4.58E 01   4.51E 01  -2.57E-07  -2.99E-02   4.37E-03    0.0      -2.34E-08  -1.57E 01
14   4.09E 01   4.55E 01   4.51E 01   3.71E-08  -2.77E-02   2.39E-03    0.0       1.77E-08  -1.57E 01
15   4.12E 01   4.52E 01   4.51E 01   3.57E-07  -2.63E-02   7.37E-04    0.0       1.52E-09  -1.57E 01
16   4.13E 01   4.51E 01   4.51E 01  -1.80E-07  -2.53E-02  -3.20E-04    0.0       2.00E-10  -1.57E 01
17   4.14E 01   4.50E 01   4.51E 01  -5.79E-07  -2.45E-02  -8.01E-04    0.0      -4.27E-09  -1.56E 01
18   4.15E 01   4.50E 01   4.51E 01  [illegible]  -2.43E-02  -1.10E-03    0.0      -1.34E-08  -1.56E 01
             DAMAGE RATE = 5.319E-06   DAMAGE = 2.747E-04
19   2.13E 01  -2.95E 03   7.57E 04  [illegible]  -1.12E-03  [illegible]   0.0      -2.17E-08  -1.57E 01
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Table H-6 Cont'd

NOT REPRODUCIBLE

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                                                                                  Table 3.4 (cont.)
                                          TIME = 2.1600000000 02

       SIGR       SIGTH       SIGZ     SIGRZ        EPR       EPTH        EPZ         EPRZ       TEM

 1   6.90E 00   1.15E 02   6.36E 01   8.16E-05 -3.13E-01   2.83E-01   0.0         1.58E-06 -4.05E 01
              DAMAGE RATE = 1.289E-03   DAMAGE = 9.593E-02
 2   2.22E 01   1.00E 02   6.39E 01   9.11E-05 -2.30E-01   2.01E-01   0.0         1.77E-06 -4.05E 01
 3   3.44E 01   8.78E 01   6.39E 01   9.59E-05 -1.62E-01   1.33E-01   0.0         1.71E-06 -4.05E 01
 4   4.16E 01   8.06E 01   6.39E 01   9.51E-05 -1.22E-01   9.25E-02   0.0         1.59E-06 -4.05E 01
 5   4.63E 01   7.59E 01   6.39E 01   9.78E-05 -9.66E-02   6.66E-02   0.0         1.66E-06 -4.05E 01
 6   4.94E 01   7.28E 01   6.39E 01   1.00E-04 -7.91E-02   4.93E-02   0.0         1.79E-06 -4.05E 01
 7   5.17E 01   7.05E 01   6.39E 01   9.31E-05 -6.67E-02   3.69E-02   0.0         1.74E-06 -4.05E 01
 8   5.34E 01   6.88E 01   6.39E 01   7.77E-05 -5.75E-02   2.75E-02   0.0         1.49E-06 -4.05E 01
 9   5.46E 01   6.77E 01   6.39E 01   5.82E-05 -5.06E-02   2.06E-02   0.0         1.11E-06 -4.05E 01
10   5.55E 01   6.67E 01   6.39E 01   4.14E-05 -4.53E-02   1.55E-02   0.0         6.57E-07 -4.05E 01
11   5.64E 01   6.59E 01   6.39E 01   3.42E-05 -4.10E-02   1.12E-02   0.0         2.53E-07 -4.05E 01
12   5.70E 01   6.53E 01   6.39E 01   1.16E-05 -3.76E-02   7.84E-03   0.0         7.06E-08 -4.05E 01
13   5.75E 01   6.48E 01   6.39E 01   3.11E-06 -3.49E-02   5.06E-03   0.0         6.56E-08 -4.05E 01
14   5.79E 01   6.44E 01   6.39E 01   1.49E-06 -3.26E-02   2.76E-03   0.0         5.97E-08 -4.05E 01
15   5.83E 01   6.40E 01   6.39E 01   1.94E-06 -3.05E-02   8.33E-04   0.0         4.03E-08 -4.05E 01
16   5.85E 01   6.38E 01   6.39E 01   1.24E-06 -2.94E-02  -4.30E-04   0.0         1.02E-08 -4.05E 01
17   5.86E 01   6.37E 01   6.39E 01   1.86E-07 -2.88E-02  -9.57E-04   0.0        -3.70E-10 -4.05E 01
18   5.87E 01   6.36E 01   6.39E 01   6.79E-07 -2.85E-02  -1.31E-03   0.0        -1.05E-08 -4.05E 01
              DAMAGE RATE = 9.170E-08   DAMAGE = 5.437E-04
19   2.99E 01  -4.07E 03   2.98E 04  -5.68E-01 -1.30E-03  -1.47E-03   0.0        -2.47E-08 -4.05E 01

                                          TIME = 2.4000000000 02

       SIGR       SIGTH       SIGZ     SIGRZ        EPR       EPTH        EPZ         EPRZ       TEM

 1   1.06E 01   1.77E 02   9.80E 01   1.32E-04 -3.56E-01   3.22E-01   0.0         2.67E-06 -6.61E 01
              DAMAGE RATE = 2.519E-03   DAMAGE = 1.913E-01
 2   3.42E 01   1.54E 02   9.85E 01   1.42E-04 -2.62E-01   2.28E-01   0.0         2.91E-06 -6.61E 01
 3   5.29E 01   1.35E 02   9.84E 01   1.58E-04 -1.85E-01   1.51E-01   0.0         2.82E-06 -6.61E 01
 4   6.41E 01   1.24E 02   9.83E 01   1.63E-04 -1.39E-01   1.03E-01   0.0         2.69E-06 -6.61E 01
 5   7.12E 01   1.17E 02   9.84E 01   1.64E-04 -1.10E-01   7.59E-02   0.0         2.75E-06 -6.61E 01
 6   7.61E 01   1.12E 02   9.84E 01   1.61E-04 -9.00E-02   5.60E-02   0.0         2.82E-06 -6.61E 01
 7   7.96E 01   1.09E 02   9.84E 01   1.47E-04 -7.59E-02   4.19E-02   0.0         2.79E-06 -6.61E 01
 8   8.21E 01   1.06E 02   9.84E 01   1.23E-04 -6.55E-02   3.15E-02   0.0         2.42E-06 -6.61E 01
 9   8.41E 01   1.04E 02   9.84E 01   9.44E-05 -5.77E-02   2.37E-02   0.0         1.80E-06 -6.61E 01
10   8.56E 01   1.03E 02   9.84E 01   6.92E-05 -5.16E-02   1.76E-02   0.0         1.26E-06 -6.61E 01
11   8.68E 01   1.02E 02   9.84E 01   5.49E-05 -4.67E-02   1.27E-02   0.0         6.07E-07 -6.61E 01
12   8.77E 01   1.01E 02   9.85E 01   2.54E-05 -4.29E-02   8.86E-03   0.0         2.24E-07 -6.60E 01
13   8.85E 01   9.98E 01   9.85E 01   1.10E-05 -3.97E-02   5.70E-03   0.0         1.62E-07 -6.60E 01
14   8.92E 01   9.92E 01   9.85E 01   5.56E-06 -3.71E-02   3.06E-03   0.0         1.29E-07 -6.60E 01
15   8.97E 01   9.87E 01   9.85E 01   4.90E-06 -3.49E-02   3.91E-04   0.0         6.11E-08 -6.60E 01
16   9.01E 01   9.83E 01   9.85E 01   3.03E-06 -3.35E-02  -5.18E-04   0.0         1.34E-08 -6.60E 01
17   9.02E 01   9.81E 01   9.85E 01   9.52E-07 -3.29E-02  -1.14E-03   0.0        -8.31E-10 -6.61E 01
18   9.03E 01   9.80E 01   9.85E 01   2.18E-06 -3.25E-02  -1.54E-03   0.0        -1.28E-08 -6.61E 01
              DAMAGE RATE = 1.825E-05   DAMAGE = 1.057E-03
19   4.58E 01  -6.21E 03   3.30E 04  -5.54E-01 -1.45E-03  -1.73E-03   0.0        -2.64E-08 -6.60E 01

                                          TIME = 2.6400000000 02

       SIGR       SIGTH       SIGZ     SIGRZ        EPR       EPTH        EPZ         EPRZ       TEM

 1  -7.84E-01  -7.18E 00  -4.14E 00  -7.78E-04  4.24E-02  -3.65E-02   0.0         2.17E-05  1.58E 02
              DAMAGE RATE = 0.0           DAMAGE = 1.916E-01
 2  -2.21E 00  -6.82E 00  -4.67E 00  -1.55E-04  3.12E-02  -2.73E-02   0.0         2.19E-05  1.58E 02
 3  -2.75E 00  -5.91E 00  -4.48E 00   1.24E-04  2.20E-02  -1.80E-02   0.0         3.31E-05  1.58E 02
 4  -3.09E 00  -5.39E 00  -4.40E 00  -1.39E-04  1.66E-02  -1.26E-02   0.0         6.01E-05  1.58E 02
 5  -3.33E 00  -5.07E 00  -4.36E 00  -6.74E-04  1.31E-02  -9.10E-03   0.0         9.34E-05  1.58E 02
 6  -3.49E 00  -4.86E 00  -4.33E 00  -5.11E-04  1.07E-02  -6.72E-03   0.0         1.20E-04  1.58E 02
 7  -3.62E 00  -4.72E 00  -4.33E 00  -4.21E-04  9.02E-03  -5.04E-03   0.0         1.31E-04  1.58E 02
 8  -3.70E 00  -4.61E 00  -4.31E 00  -4.55E-04  7.79E-03  -3.60E-03   0.0         1.29E-04  1.55E 02
 9  -3.77E 00  -4.53E 00  -4.31E 00  -3.72E-04  6.87E-03  -2.30E-03   0.0         1.13E-04  1.55E 02
10  -3.83E 00  -4.47E 00  -4.31E 00  -1.19E-04  6.15E-03  -2.13E-03   0.0         8.92E-05  1.55E 02
11  -3.87E 00  -4.42E 00  -4.30E 00   9.46E-05  5.58E-03  -1.55E-03   0.0         6.39E-05  1.55E 02
12  -3.91E 00  -4.39E 00  -4.30E 00   1.24E-04  5.13E-03  -1.07E-03   0.0         4.28E-05  1.55E 02
13  -3.94E 00  -4.36E 00  -4.30E 00   7.94E-05  4.77E-03  -7.06E-04   0.0         2.80E-05  1.55E 02
14  -3.96E 00  -4.33E 00  -4.30E 00   3.90E-05  4.47E-03  -3.76E-04   0.0         1.80E-05  1.55E 02
15  -3.98E 00  -4.31E 00  -4.30E 00   5.42E-05  4.21E-03  -1.23E-04   0.0         1.11E-05  1.55E 02
16  -3.99E 00  -4.29E 00  -4.30E 00   7.07E-05  4.05E-03   4.06E-05   0.0         6.23E-06  1.55E 02
17  -4.00E 00  -4.29E 00  -4.30E 00   5.49E-05  4.01E-03   1.22E-04   0.0         2.96E-06  1.55E 02
18  -4.00E 00  -4.29E 00  -4.30E 00   2.23E-05  3.95E-03   1.71E-04   0.0         9.26E-07  1.55E 02
              DAMAGE RATE = 0.0           DAMAGE = 1.074E-03
19  -2.18E 00   2.98E 02  -4.17E 03   2.21E-01  1.31E-04   1.94E-04   0.0         9.62E-09  1.55E 02
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Table H-6 Cont'd

APPENDIX H

Table 5.4 (cont.)

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                                      TIME = 2.4000000000D 02

     SIGR       SIGTH       SIGZ       SIGRZ        EPR        EPTH         EPZ         EPRZ        TEM

 1 -3.44E-01   1.94E-01  -6.06E-02  -7.56E-04   2.60E-04  -2.35E-04   0.0          2.88E-05   1.35E 02
             DAMAGE RATE = 4.570E-21   DAMAGE = 1.710E-31
 2 -7.49E-01  -4.03E-01  -5.83E-01  -8.80E-05   1.98E-04  -1.66E-04   0.0          3.04E-05   1.35E 02
 3 -5.47E-01  -2.80E-01  -3.99E-01   2.08E-04   1.42E-04  -1.10E-04   0.0          4.57E-05   1.35E 02
 4 -4.49E-01  -2.34E-01  -3.10E-01  -9.92E-05   1.08E-04  -7.56E-05   0.0          7.56E-05   1.35E 02
 5 -3.65E-01  -2.17E-01  -2.70E-01  -5.02E-04   8.35E-05  -5.26E-05   0.0          1.21E-04   1.35E 02
 6 -3.28E-01  -2.11E-01  -2.55E-01  -5.81E-04   6.95E-05  -3.73E-05   0.0          1.53E-04   1.35E 02
 7 -3.04E-01  -2.14E-01  -2.40E-01  -4.97E-04   5.81E-05  -2.73E-05   0.0          1.67E-04   1.35E 02
 8 -2.90E-01  -2.11E-01  -2.30E-01  -5.21E-04   4.96E-05  -2.00E-05   0.0          1.63E-04   1.35E 02
 9 -2.81E-01  -2.15E-01  -2.34E-01  -4.16E-04   4.19E-05  -1.46E-05   0.0          1.43E-04   1.35E 02
10 -2.74E-01  -2.18E-01  -2.32E-01  -1.25E-04   3.63E-05  -1.05E-05   0.0          1.13E-04   1.35E 02
11 -2.69E-01  -2.21E-01  -2.31E-01   1.15E-04   3.19E-05  -7.24E-06   0.0          8.05E-05   1.35E 02
12 -2.65E-01  -2.23E-01  -2.30E-01   1.54E-04   2.80E-05  -4.73E-06   0.0          5.39E-05   1.35E 02
13 -2.62E-01  -2.25E-01  -2.30E-01   1.03E-04   2.45E-05  -2.72E-06   0.0          3.50E-05   1.35E 02
14 -2.60E-01  -2.27E-01  -2.30E-01   5.56E-05   2.12E-05  -1.09E-06   0.0          2.23E-05   1.35E 02
15 -2.58E-01  -2.29E-01  -2.30E-01   6.67E-05   1.59E-05   7.72E-08   0.0          1.27E-05   1.35E 02
16 -2.56E-01  -2.30E-01  -2.29E-01   8.00E-05   1.40E-05   7.85E-07   0.0          7.71E-06   1.35E 02
17 -2.56E-01  -2.30E-01  -2.30E-01   6.64E-05   1.87E-05   1.07E-06   0.0          3.71E-06   1.35E 02
18 -2.55E-01  -2.31E-01  -2.30E-01   2.43E-05   1.64E-05   1.30E-06   0.0          1.16E-06   1.35E 02
             DAMAGE RATE = 0.0         DAMAGE = 1.073E-03
19 -2.08E-01   2.49E 01  -7.36E 00   1.15E-01   3.16E-07   1.40E-06   0.0          5.02E-09   1.35E 02
```

Table H-6 Cont'd

APPENDIX I

NON-LINEAR ANALYSIS BASED ON PROPELLANT DILATATION

Appendix I

NON-LINEAR ANALYSIS BASED ON PROPELLANT DILATATION

In general, the response of grains to various loading conditions is mainly determined by the dilatational behavior of the propellant, hence, it would appear that accounting for the non-linear dilatational response of propellant is of primary importance.

Although insufficient experimental data is available to permit a comprehensive characterization of the non-linear dilatational effects, it was felt that an evaluation could be obtained by utilizing a description which qualitatively accounts for the behavior reported for a limited class of stress states in References I-1 and I-2, and which predicts reasonable results for other stress states. Because of the tentative nature of the characterization, it was deemed desirable to approximate the non-linear behavior by an equation which could be incorporated into the existing analysis with a minimum amount of effort.

With the above objectives in mind the following approximations were made: (1) The thermo-rheologically simple linear viscoelastic distortional stress-strain law and the temperature induced volume change relationships would remain unchanged; (2) the stress induced dilatational response would be approximated as elastic. It was felt that the viscoelastic dilatation (compare Figures 11 and 12 of Reference I-2), is of secondary importance as compared to the basic non-linear effects. Additionally, the lack of experimental evidence concerning dilatation during unloading precludes its consideration at this time. (3) The stress-dilatation relationship would be considered to be temperature independent (it was felt that the actual temperature dependence was of secondary importance). (4) The characterization would be limited to relatively small strains. (5) The elastic relationship between the dilatation and the stress state could be expressed as:

$$(\theta - 3\bar{\alpha}\Delta T) \overset{\sim}{=} f(g(\Theta_i)) = f(\tau_{ij}) \qquad \text{(I-1)}$$

where "f" is a non-linear algebraic function of the quantity "g", $g(\Theta_i)$ is in turn a non-linear function of the stress invariants. The stress invariants are defined by the equations

$$\Theta = \tau_{11} + \tau_{22} + \tau_{33} \qquad \text{(I-2)}$$

$$\Theta_2 = \begin{vmatrix} \tau_{22} & \tau_{23} \\ \tau_{32} & \tau_{33} \end{vmatrix} + \begin{vmatrix} \tau_{11} & \tau_{31} \\ \tau_{31} & \tau_{33} \end{vmatrix} + \begin{vmatrix} \tau_{11} & \tau_{12} \\ \tau_{12} & \tau_{22} \end{vmatrix}$$

$$\Theta_3 = \begin{vmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{12} & \tau_{22} & \tau_{23} \\ \tau_{13} & \tau_{23} & \tau_{33} \end{vmatrix}$$

For linear theory we have $g = \Theta$ and $f = \frac{K}{3} g$ i.e.

$$(\theta - 3\alpha\Delta T) = K \frac{\Theta}{3} \qquad \text{(I-3)}$$

Appropriate forms of the functions "f" and "g" were determined in the following manner. For a uniaxial state of stress (i.e., $\tau_{ij} = 0$ except $\tau_{11} \neq 0$) noting that $\Theta_2 = \Theta_3 = 0$, it was assumed that

$g(\Theta_i) \approx \Theta = \tau_{11}$, i.e.,

$$(\theta - 3\alpha\Delta T) \approx f(\tau_{11}) \qquad \text{(I-4)}$$

Inspecting results from uniaxial tests (and uniaxial tests with small superimposed hydrostatic pressures) the form of the function "f" was determined. The form of the function "g" was determined by inspecting the results of uniaxial tests with various levels of superimposed hydrostatic pressures. Lastly, the forms of the functions "f" and "g" were appropriately modified so that the predictions for other stress states would intuitively agree with the physical explanation given in Reference (I-2) for the non-linear dilatation phenomenon.

The experimental results presented in Figures 8, 9, 10, 12 and 14 of Reference (I-2) were plotted in the form $\theta \sim \tau_{11}$ in Figure I-1. Although these data certainly demonstrate a dependence upon temperature and strain rate (and of course a dependence upon "volume of loading"), insufficient data were available to permit a description of this dependence, hence, it was neglected. No data was available for compressive stress states, however, it was anticipated that for negative stresses the relationship would be relatively linear. The data presented in Figure I-1 (i.e., the function "f") was approximated by a hyperbola, i.e., the relationship between θ and the function "g" was written as:

$$\theta = \frac{P_2 + P}{2} + \frac{\beta_2 - \beta}{2} + B \qquad \text{(I-5)}$$

where

$$\beta_1 = \theta_o + \frac{1}{K_T} (g_i - g_o)$$

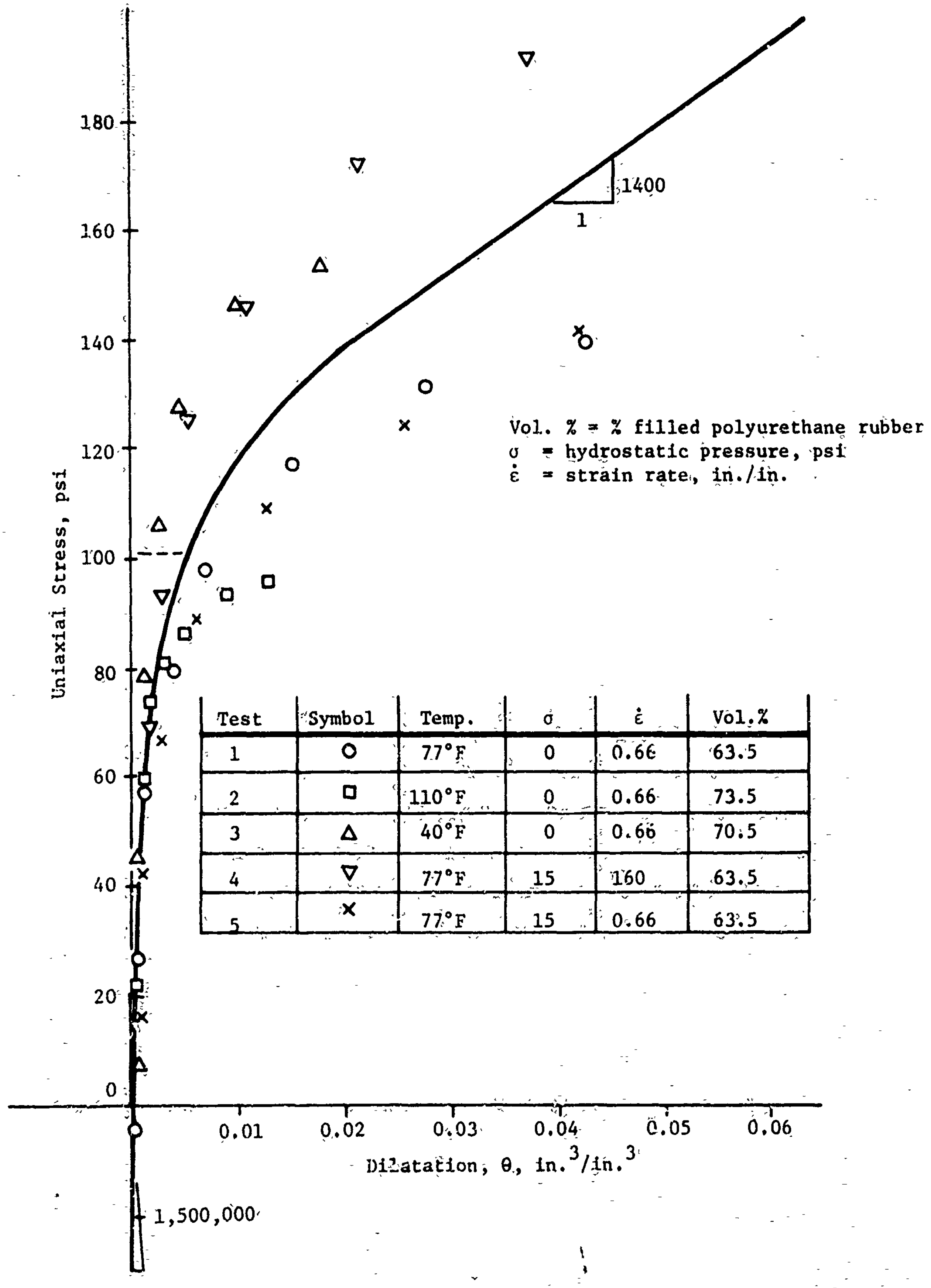
$$B = \frac{\beta_1 (\theta_o - g_o/K_c)}{1 - \frac{\beta_1 K_g}{g_i}}$$

$$\beta_2 = \frac{BK_T}{g_i} - \frac{g}{K_c}$$

$$\beta = \frac{1}{K_T} (g_i - g)$$

The meaning of the parameters K_T, K_c, g_i, g_o and θ_o is illustrated in

VOLUME CHANGE
(Data taken from Reference (7))



Vol. % = % filled polyurethane rubber
σ = hydrostatic pressure, psi
ε̇ = strain rate, in./in.

Test	Symbol	Temp.	σ	ε̇	Vol.%
1	○	77°F	0	0.66	63.5
2	□	110°F	0	0.66	73.5
3	△	40°F	0	0.66	70.5
4	▽	77°F	15	160	63.5
5	×	77°F	15	0.66	63.5

Figure I-1

Figure I-2. The form of the function g $(|\theta|_1)$ was determined by considering the data presented in Figure 1 of Reference (I-2). A number of different functional forms were investigated from among which the following form was selected.

$$g(|\theta|_1) = |\theta| - K_1 |\theta|_2 + K_2 |\theta|_3 + K_3 |\theta|^3 \qquad \text{(I-6)}$$

The values of K_1, K_2 and K_3 were selected by noting that Equation (I-1) states that for a given amount of dilatation the value of "g" should be a constant. Thus for the different stress states (corresponding to different values of hydrostatic pressure) given in Figure 1 of Reference (I-2) an attempt was made to select the function "g" so that for a given value of θ all states would yield a common value of "g". The following values were found for the parameters

$$K_1 = 4.9 \times 10^{-3}$$

$$K_2 = 2.3 \times 10^{-5}$$

$$K_3 = 5.7 \times 10^{-6}$$

Using these parameters the following table was constructed

1	2	3	4	5
Hydrostatic Pressure, psi	θ = .005		θ = .01	
	g	θ/3	g	θ/3
15	66	54	84	68
65	66	-5	99	14
165	101	-150	125	-115

If the dilatational response could be described by a linear elastic law all the entires in Column 3 (and in Column 5) would need to be approximately equal, the extreme grossness of this approximation is evident. If the non-linear function "g" is to represent the experimental data all the entires in Column 2 (and in Column 4) would need to be

PARAMETERS DEFINING HYPERBOLIC
DILATATIONAL RELATIONSHIP

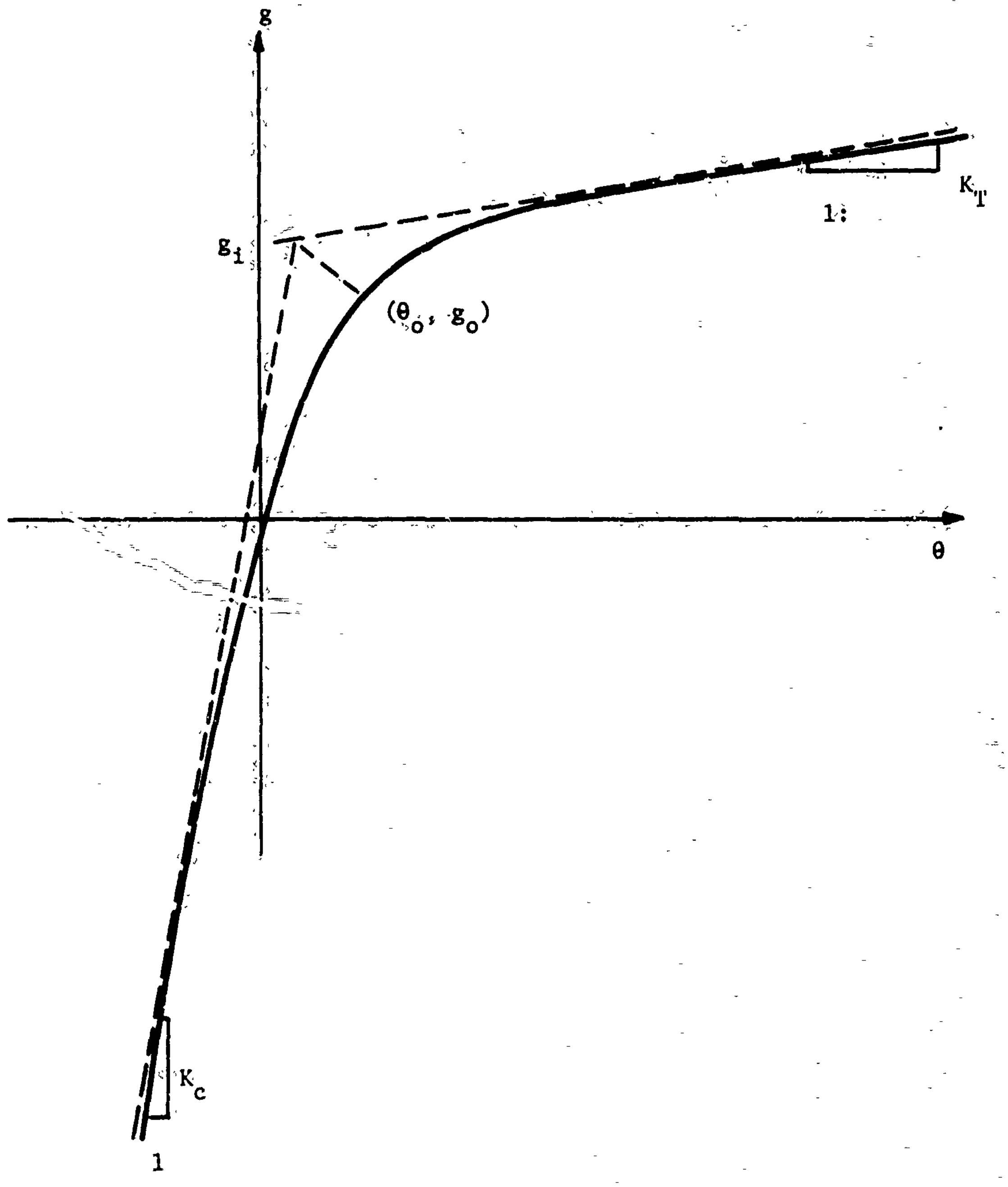

g
g_i
(θ_o, g_o)
K_T
1
θ
K_c
1

Figure I-2

approximately equal, while this is not true, the very substantial improvement of the non-linear representation as compared to the linear representation is evident.

While no experimental information was available for stress states other than the uniaxial test with superimposed hydrostatic pressure, it is extremely important that the proposed model predicts reasonable results for other stress states as these stress states may occur in a given grain problem. Utilizing the proposed non-linear model dilatation was predicted for the following stress states:

(a) Uniaxial stress with superimposed hydrostatic pressure (Figure I-3)

(b) Hydrostatic pressure (Figure I-4)

(c) Biaxial stress with superimposed hydrostatic pressure (Figure I-5)

(d) Simple shear with superimposed hydrostatic pressure (Figure I-6)

The straight lines in Figure I-3 indicate the predictions of the linear model, the curved lines the predictions of the non-linear model. The predictions of the non-linear model qualitatively appear to be reasonable.

The incorporation of the non-linear dilatational relationship (Equation (I-1)) into the existing thermoviscoelastic analyses presented some difficulties. The existing thermoviscoelastic analysis employs the following incremental relationship for dilatation (for time increment N)

$$(\Delta\theta_N - 3\alpha\Delta T_N) = K_N \frac{\Delta\theta_N}{3} + \chi_N \quad t_{N-1} \leq t \leq t_N \qquad (I-7)$$

In the previous linear analysis K_N was considered to be a constant (the elastic bulk modulus) and $\chi_N = 0$. The utilization of Equation (I-7) as an approximation to Equation (I-1) was accomplished as follows:

Having obtained the solution for time t_{N-1} consider the utilization of Equation (I-7) for $t_{N-1} - t_N$. Symbolically denoting the stress state at time t_{N-1} as $\tau_{ij_{N-1}}$ and the incremental change in stress state for $t_{N-2} - t_{N-1}$ as $\Delta\tau_{ij_{N-1}}$. The following two associated stress states are defined:

DILATATIONAL PREDICTIONS FOR UNIAXIAL STRESS WITH SUPERIMPOSED HYDROSTATIC PRESSURE

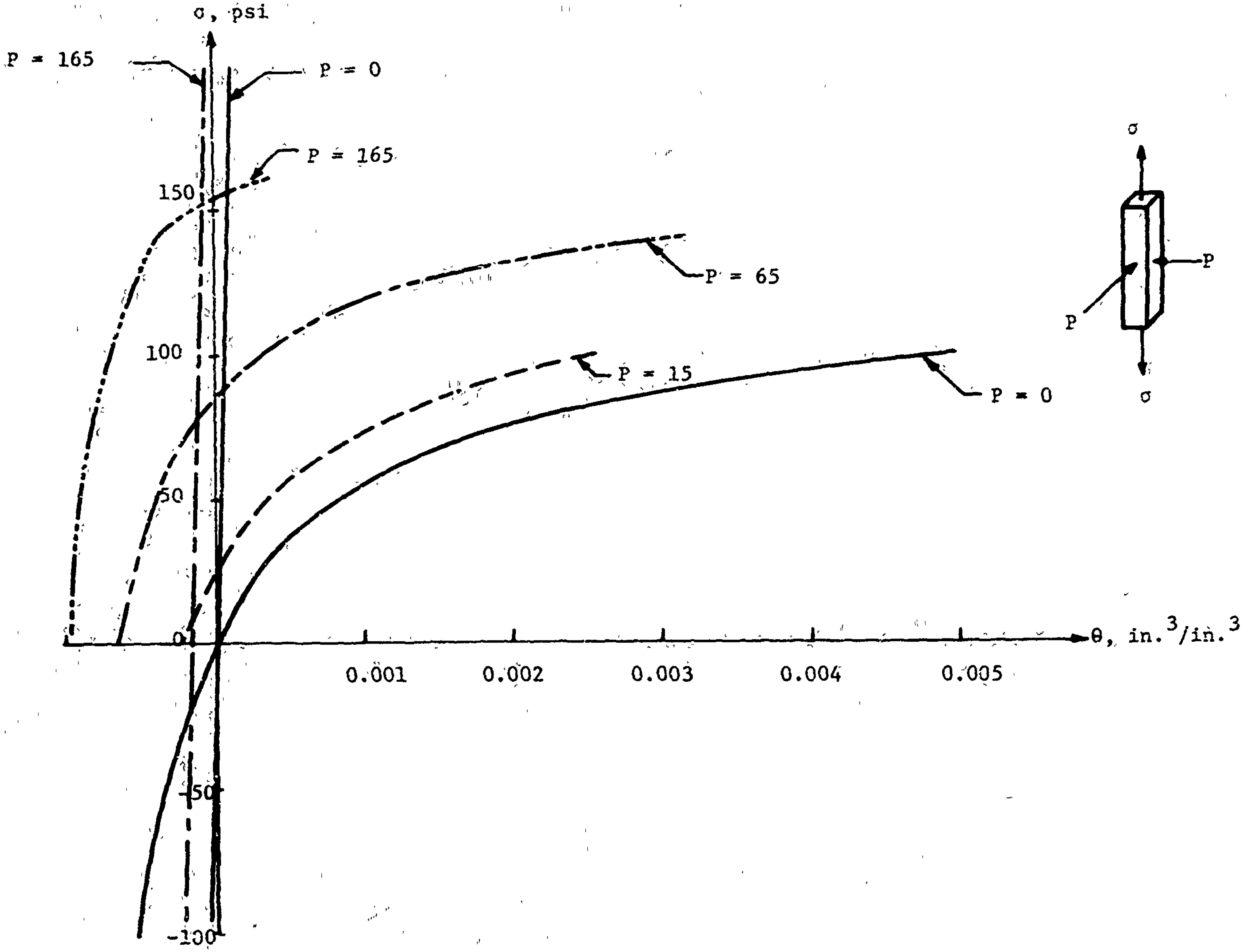


Figure I-3

DILATATION PREDICTIONS FOR HYDROSTATIC PRESSURE

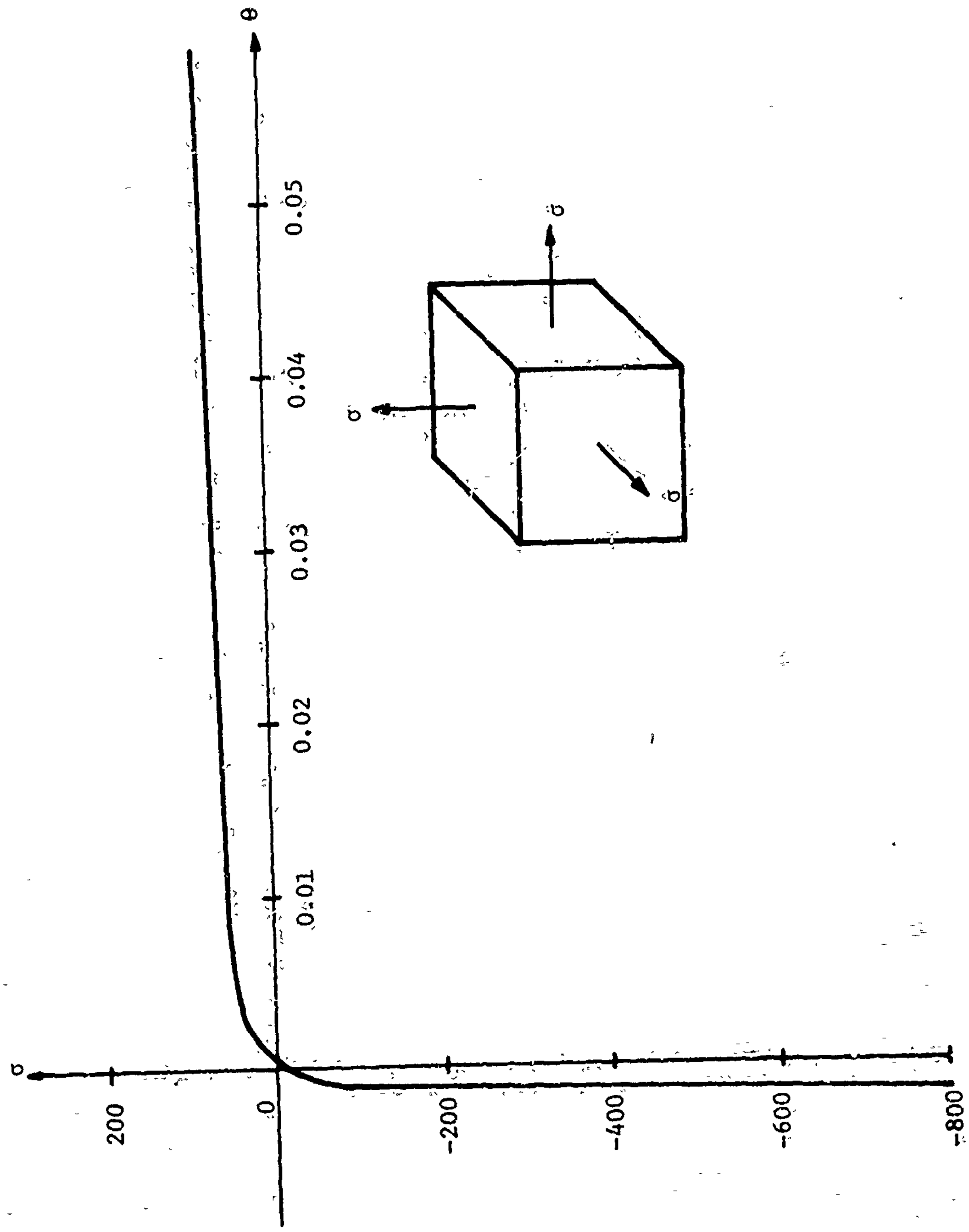
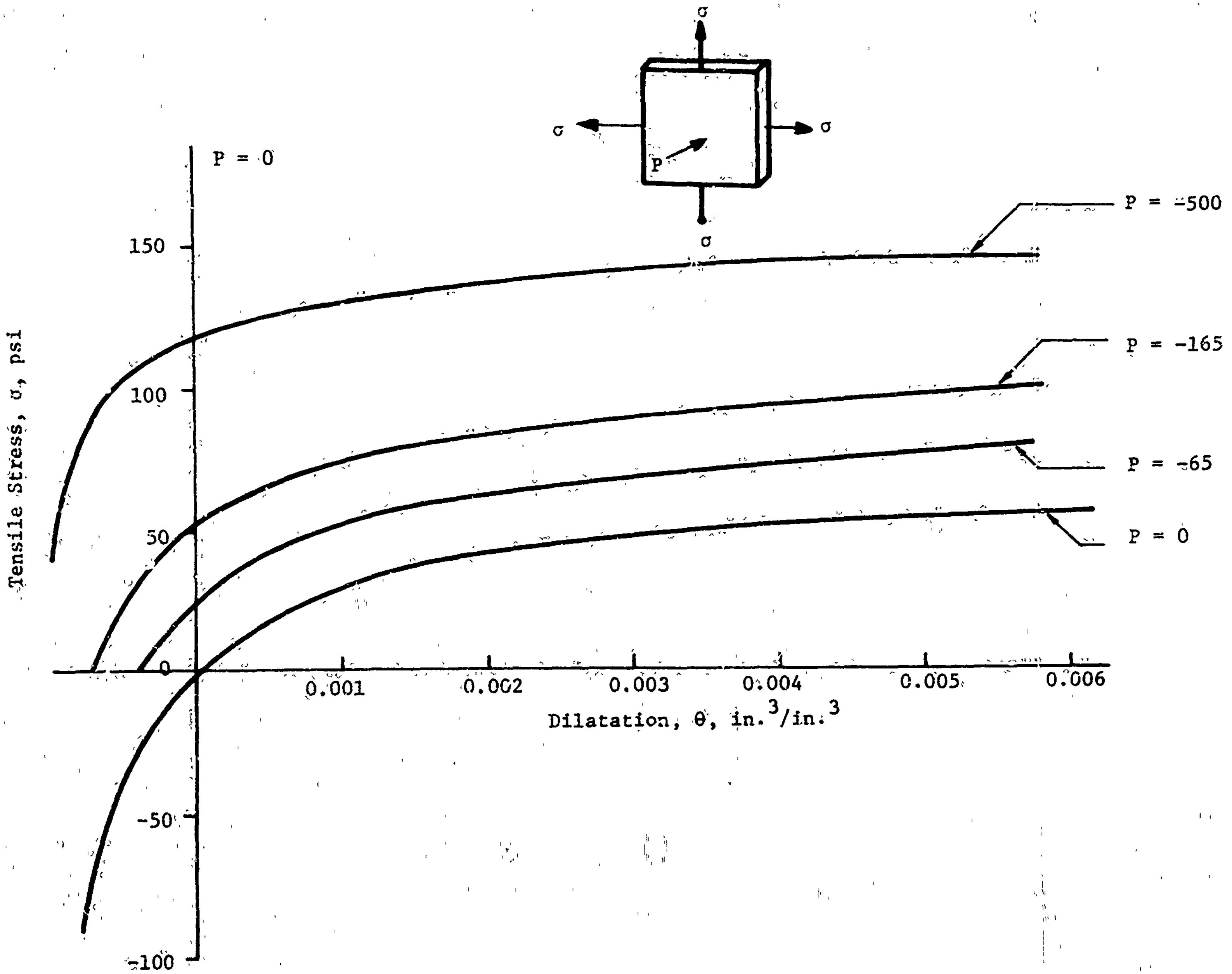


Figure I-4



DILATATIONAL PREDICTIONS FOR BIAXIAL STRESS WITH SUPERIMPOSED HYDROSTATIC PRESSURE

Figure I-5

DILATATIONAL PREDICTIONS FOR SHEAR WITH SUPERIMPOSED HYDROSTATIC PRESSURE

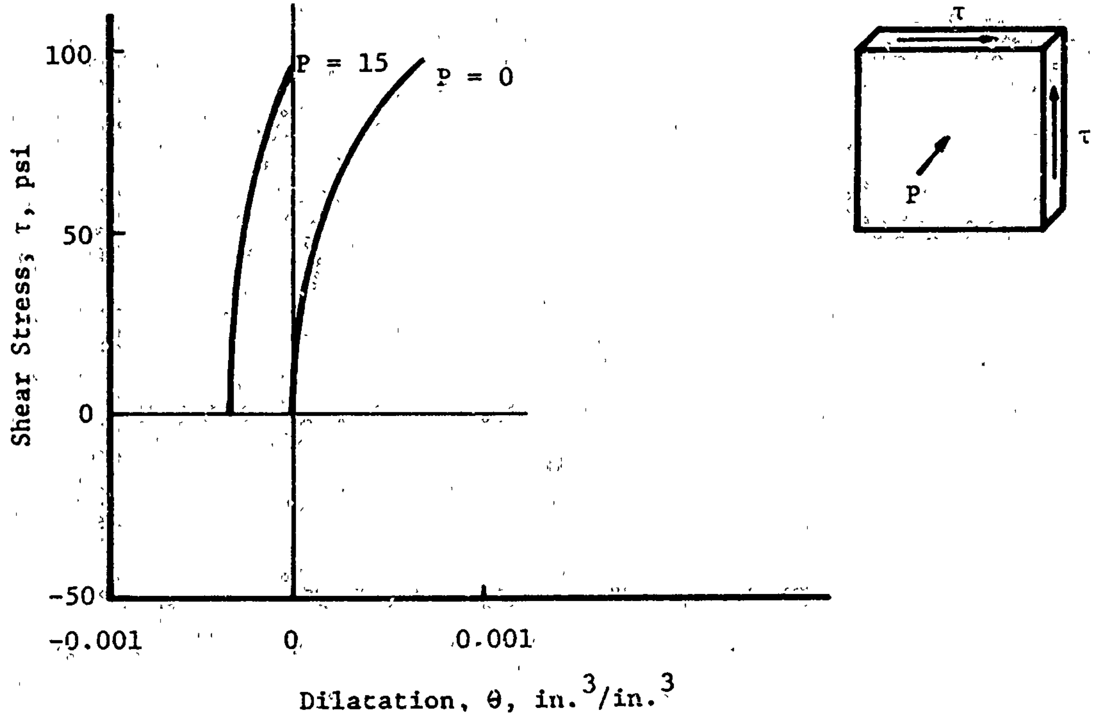


Figure I-6

$$\tau_{ij_a} \equiv \tau_{ij_{N-1}} - \Delta\tau_{ij_{N-1}} \qquad \text{(I-8)}$$

$$\tau_{ij_b} \equiv \tau_{ij_{N-1}} + \Delta\tau_{ij_{N-1}} \qquad \text{(I-9)}$$

Utilizing Equations (I-1), (I-2), (I-8), and (I-9), the following quantities are calculated θ_a, θ_b, Θ_a and Θ_b. The following approximate relationship is written for values of θ between θ_a and θ_b

$$\theta = \frac{\Theta_a \theta_b - \Theta_b \theta_a}{\Theta_a - \Theta_b} + \frac{\theta_a - \theta_b}{\Theta_a - \Theta_b} \Theta \qquad \text{(I-10)}$$

Assuming that the absolute value of the change in stress during the increment Δt_N will be of the same order of magnitude as $|\Delta\tau_{ij_{N-1}}|$

(i.e., using Equation (I -10) and writing $\theta_N = \theta_{N-1} + \Delta\theta_N$ and $\Theta_N = \Theta_{N-1} + \Delta\Theta_N$ yields

$$\theta_{N-1} + \Delta\theta_N = \frac{\Theta_a \theta_b - \Theta_b \theta_a}{\Theta_a - \Theta_b} + \frac{\theta_a - \theta_b}{\Theta_a - \Theta_b} (\Theta_{N-1} + \Delta\Theta_N) \qquad \text{(I-11)}$$

Comparing the above expression to Equation (I-7) yields

$$K_N = \frac{\theta_a - \theta_b}{\Theta_a - \Theta_b}$$

$$X_N = \frac{\Theta_a \theta_b - \Theta_b \theta_a}{\theta_a - \Theta_b} + \frac{\theta_a - \theta_b}{\Theta_a - \Theta_b} \Theta_{N-1} - \theta_{N-1}$$

Because of the secant approximation to the non-linear function, in general, $X_N \neq 0$.

Appendix I

References

I-1 Farris, R. J., "The Character of the Stress-Strain Function for Highly Filled Elastomers", Trans. of the Soc. of Rheology 12:2, 303-314 (1968).

I-2 Farris, R. J., "The Influence of Vacuole Formation on the Response and Failure of Filled Elastomers", Trans. of the Soc. of Rheology 12:2, 315-334 (1968).

APPENDIX J

BASIC CUMULATIVE DAMAGE EQUATIONS

APPENDIX J

BASIC CUMULATIVE DAMAGE EQUATIONS

The LCD relation merely states the manner in which damage can be added. To apply the relation in a three-dimensional stress field requires a time-dependent, stress failure criterion. An empirical approach showed that the applicable relation is a time-dependent, maximum principal stress, failure criterion (MPS).

Combining the LCD and MPS criteria leads to integral relations which take full account of the past stress-time-temperature history at a point in the grain.

1. Linear Cumulative Damage Relations

The very simple relation, applicable to solid propellant failures, is defined in terms of tests made under a constantly imposed "true" stress, σ_t. For a number, N, of discrete stress levels the accumulated damage fraction, ΣD, is given by the following linear relation:

$$\Sigma D = \frac{1}{P(n)} \sum_{i=1}^{N} \frac{\Delta t_i}{\bar{t}_{fi}} \qquad \text{(J-1)}$$

where ΣD is the cumulative damage

$P(n)$ is a statistical distribution parameter and relates the nth test specimen in the distribution to the mean of the population.

Δt_i is the increment of time the specimen is exposed to the ith "true" stress level

$\bar{t}_{fi}$ is the mean time-to-failure for the population of specimens if the specimens saw only the ith "true" stress level.

The accumulated damage, ΣD, thus gives the fraction of that damage required to fail the specimen. Thus, by definition $\Sigma D = 1$ at failure.

The parameter, $P(n)$, is a highly useful term in that it provides the focal point for all the statistical studies pertinent to the cumulative damage relations. In its simplest form $P(n)$ defines the position of the indiviaul failure with respect to the mean of the distribution of these failures. This is seen on considering specimens held under a single stress, where $P(n)$ equals the ratio of the time-to-failure, $t_f(n)$, for the nth individual divided by the mean time-to-failure for the entire population, $\bar{t}_f$.

Thus, we have

$$P(n) = t_f(n)/\overline{t}_f \qquad \text{(J-2)}$$

The cumulative damage relation merely preserves this concept of P(n) for conditions of complicated loading histories.

In Reference J-1, it was shown that P(n) is independent of the stress level. This justifies placing the parameter outside the summation in Equation (J-1).

Statistical evaluations of P(n) obtained from solid propellant failure testing, show the parameter to follow approximately a log normal distribution.

2. Time-Dependent Maximum Principal Stress Failure Criterion

The most general form of the MPS failure criterion for solid propellants appears to be a modified power-law relation. However, a simple power-law approximation to this criterion covers all of the practical motor problems that we have met to date. This power-law relation has the following form:

$$\overline{t}_f = t_o a_T \left(\frac{\sigma_t - \sigma_{cr}}{\sigma_{t_o} - \sigma_{cr}} \right)^{-B} \qquad \text{(J-3)}$$

where

σ_t is the "true" stress applied to the specimen.

$\overline{t}_f$ is the mean time-to-failure of specimens held under the constant true stress, σ_t.

t_o is the unit value of the time for whatever units are used in measuring t_f.

σ_{to} is the true stress required to fail the specimen in the time t_o.

σ_{cr} is the critical true stress below which no failures are observed.

a_T is the time-temperature shift relation

It has been found to apply to solid propellant behavior with correlation coefficients usually exceeding 0.98.

The limiting stress, σ_{cr}, is difficult to evaluate because it is usually not large compared with the relatively large variability typical of solid propellants. It appears to be negligibly small (except for the effects of pressure) for all the propellants tested to date.

The cumulative damage relation is generalized as follows. First, Equation (J-3) is combined with Equation (J-1) to obtain the cumulative damage relation in terms of discrete stress levels, loading time intervals, and test temperatures.

$$\Sigma D = \frac{1}{P(n)} \Sigma \left[\frac{\sigma_{ti} - \sigma_{cr}}{\sigma_{to} - \sigma_{cr}}\right]^{B} \frac{\Lambda_{ti}}{t_o a_T} \qquad \text{(J-4)}$$

where σ_{ti} is the ith stress level.

For continuous changes in the stress and in the temperature, Equation (J-4) becomes:

$$\Sigma D = \frac{1}{P(n)\ (\sigma_{to} - \sigma_{cr})^{B}\ t_o} \int_o^t \frac{(\sigma_t - \sigma_{cr})^{B}}{a_T(t)}\, dt \qquad \text{(J-5)}$$

where σ_t, the true stress, is now a function of time.

$a_T(t)$ is the time-temperature shift relation with the temperature expressed as a function of time.

Equation (J-5) represents a general form of the linear cumulative damage relation for solid propellants. This equation permits the summation of damage for any type of thermal or mechanical loading history, provided the stresses, times and temperatures are known. Also, Equation (J-5) can be applied without change to three dimensional stress problems.

REFERENCES

J-1 Bills, K. W., Jr., Peterson, F. E., Steele, R. D., and Sampson, R. C., "Development of Criteria for Solid Propellant Screening and Preliminary Engineering Design", Aerojet Report 1159-81F (December 1968).

APPENDIX K

STUDY OF PROPELLANT FAILURE UNDER PRESSURE

APPENDIX K

STUDY OF PROPELLANT FAILURE UNDER PRESSURE

A. FAILURE MECHANISMS

The basic mechanisms of propellant failure have been extensively studied under current and past research programs$^{(K-1, K-2)}$. Failure is considered to be a three step process beginning with vacuole formation. The vacuoles may initiate in the polymer or at the binder-filler interface. The associated volume change may be so negligible that the process is termed a "Mullin's Effect". The second step is the extension of the vacuoles until they overlap producing a thin membrane of polymer between them. The third and final step is the production of a critical, biaxial, stress field in one or more of the thin membranes. The resulting tears propagate very slowly relative to the speed of sound in the propellant. This failure mechanism is the basis of the considerations made later.

The maximum principal stress failure criterion, Equation (J-3), contains a simple stress difference term, $\sigma_t - \sigma_{cr}$. Some feel that this indicates a maximum shear stress criterion to be operative. In biaxial tension this is expressed by

$$\tau(\text{critical}) = (\sigma_1 - \sigma_2)/2 \qquad \text{(K-1)}$$

where τ(critical) is the maximum shear strength of the material

σ_1 and σ_2 are principal stresses

Considering the stress difference terms only, for both failure criteria, leads to the tabulation in Table K-1. It is assumed that a maximum principal stress, σ, is imposed in tension. The remaining stresses are tabulated.

Table K-1 shows that the two criteria give identical results for simple uniaxial and strip-biaxial tensile data from which the MPS criterion was derived. However, the two criteria differ greatly when applied to the triaxial, tensile, stress field of the poker chip specimen. In this case, the stress difference for the maximum shear stress criterion is dependent upon 1-k, which is very small, indicating that very large axial stresses, σ, would be required to fail the specimen. On the other hand, the MPS criterion gives a relatively large value for the stress difference term, $\sigma_t - \sigma_{cr}$. This difference, like those for the uniaxial and biaxial tensile data, equals the axial stress on the specimen. Tests on poker chip specimens showed the MPS criterion to hold,$^{(K-2)}$ while the maximum shear stress criterion is grossly in error.

TABLE K-1

STRESS DIFFERENCES AT ATMOSPHERIC PRESSURE

Test Mode	MFS Criterion			Maximum Shear Stress Criterion		
	σ_t	σ_{cr}*	$\sigma_t - \sigma_{cr}$	σ_1	σ_2	$\sigma_1 - \sigma_2$
Uniaxial Tensile Test	σ	0	σ	σ	0	σ
Strip-Biaxial Tensile Test	σ	0	σ	σ	0	σ
Poker Chip Tensile Test	σ	0	σ	σ	$k\sigma$	$(1-k)\sigma$

---

* σ_{cr} is set equal to zero, which appears to be the case for many composite propellants.

** The maximum tensile stress at the center of the specimen.

*** k is a proportionality factor which, in this case, is close to one.

This result is not surprising in view of the fialure mechanisms summarized above. Tensile failures in uniaxial, biaxial, and poker chip specimens follow the same mechanism. Once the vacuoles are produced all three test modes become equivalent, in that failure is produced by extension of a foam-like material until the biaxial stress in the membranes exceeds the critical value and tearing starts.

B. EVALUATING THE TIME-PRESSURE SHIFT FACTOR

The time-pressure shift factor for propellants is best understood on referring to the foam-like failure behavior mentioned above. Before the vacuoles are formed, or when they are still quite small, a_p equals one (up to about 1500 psi pressure). This was shown for tensile moduli by Wiegand,$^{(K-3)}$ Hazelton$^{(K-4)}$ and Lim and Tshoegel$^{(K-5)}$. At this point, the a_p values are those for the solid binder which requires large pressure changes to effect significant changes in a_p.$^{(K-6 \text{ to } 14)}$ On the other hand, propellant failure data are sensitive to relatively small pressure changes. Dilatation constitutes the only physical difference in the propellant between the points where initial tensile moduli and failure data are obtained. At failure, this dilatation is extensive and can be partially suppressed by superimposed hydrostatic pressures. As shown by Farris$^{(K-15)}$ and Lim and Tschoegel$^{(K-5)}$, the dilatation at failure is positive, even at the higher pressure levels.

Taken together these observations indicate that a_p provides a measure of the suppression of vacuole formation. But, σ_{cr} is also a measure of the same effect. That is, σ_{cr} is the stress level at which dilatation is completely suppressed; or, conversely, the stress level where vacuoles are first formed. Also, σ_{cr} is pressure dependent, defining the point of vacuole formation over the pressure range. Thus, a_p must be a simple function of σ_{cr}. That is,

$$a_p = f\,(\sigma_{cr}) \qquad \text{(K-2)}$$

Equation (K-2) tells us how to evaluate a_p deep in the grain. The critical stress acts in the same direction as the maximum principal stress, σ_t, and it includes that portion of the inner-bore pressure that is effective deep within the grain and which acts in the same direction as σ_t. The fraction, f, of the inner-bore pressure that acts within the grain can be obtained directly from existing stress analyses. Using this fraction the critical stress becomes, on using Equation (14) of the text,

$$\sigma_{cr} = (F/A)_{cr} - fP_i \qquad \text{(K-3)}$$

where P_i is the inner-bore pressure

The appropriate values of a_p for use in Equation (K-2) are obtained from empirically obtained curve of a_p versus pressure, the desired value of a_p read from the curve at a pressure numerically equal to σ_{cr}, the latter being obtained from Equation (K-3).

REFERENCES

K-1 Bills, K. W., Jr., Campbell, D. M., Sampson, R. C., and Steele, R. D., "Failures in Grains Exposed to Rapid Changes of Environmental Temperatures", Aerojet Report 1236-81F (Contract No. N00017-68-C-4415) (April 1969).

K-2 Bills, K. W., Jr., et. al, "Solid Propellant Cumulative Damage Program", Final Report No. AFRPL-TR-63-131, Contract No. F04611-67-C-0102, (October 1968).

K-3 Wiegand, J. H., "Study of Mechanical Properties of Solid Propellants", Report No. 0411-10F, Aerojet-General Corporation, 1962.

K-4 Hazelton, I. G., and Planck, R. W., "Propellant Characterization for Firing and Flight", Bulletin of 45th Meeting of ICRPG Working Group on Mechanical Behavior, p. 287 (Nov. 16-19, 1965).

K-5 Lim, C. K., and Tschoegel, N. W., "The Effect of Pressure on the Mechanical and Ultimate Properties of Filled Elastomers", Bulletin of the Eighth Meeting of the JANNAF Mechanical Behavior Working Group, p. 153 (March 1970).

K-6 Hughes, D. S., et al, J. App. Phys. 21, 294 (1950).

K-7 Masuoka, S., and Maxwell, B., J. Poly. Sci, 32, 131 (1958).

K-8 J. Res. Nat. Bur. Stds., 50, 311 (1953).

K-9 J. Appl. Phys., 30, 337 (1959).

K-10 Bull. Amer. Phys. Soc. II, 5, 203 (1960).

K-11 Modern Plastics, 35, 174 (1957).

K-12 Trans. Soc. Rheol., 4, 347 (1960).

K-13 J. Chem. Phys., 26, 196 (1957).

K-14 Proc. Roy. Soc., A253, 52 (1959).

K-15 Farris, R. J., "The Influence of Vacuole Formation on the Response and Failure of Filled Elastomers", Trans. of the Soc. of Rheology, 12:2, 315-334 (1968).

APPENDIX L

EFFECTS OF PREVIOUS DAMAGE

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EFFECTS OF PREVIOUS DAMAGE

The cumulative damage relations can be used to evaluate the effects of previous damage upon propellant failure data. This may be done using the LCD analysis of a single test-to-failure curve which duplicates a given failure condition in the motor. When considering the inner-bore failure of a grain on motor pressurization we use a high rate, uniaxial tensile test performed under a superimposed hydrostatic pressure.

The cumulative damage relation appropriate to this analysis is derived here, starting with Equation (16) from the text. For this analysis, considerations are simplified by assuming the pressure and temperature to be held constant. Thus a_p and a_T become constants and may be taken outside the integral in Equation (16).

For the previously undamaged specimen the time-to-break, t_b, and the maximum true stress become normalizing parameters for the integral term, as originally shown in Reference (L-1). Thus, we let

$$A = \int_0^{t_b/t_b} \frac{(\sigma_t - \sigma_{cr})^B}{(\sigma_{tM} - \sigma_{cr})^B} \, d\,(t/t_b) \qquad \text{(L-1)}$$

Equation (16) after normalization by substituting A, gives

$$\Sigma D = 1 = \frac{(\sigma_{tM} - \sigma_{cr})^B \; t_b A}{P(n) \; (\sigma_{to} - \sigma_{cr})^B \; t_o a_T a_p} \qquad \text{(L-2)}$$

Prior to failure we can use the same normalization terms, t_b and $(\sigma_{tM} - \sigma_{cr})^B$, but take the quantity A prior to failure with the upper integration limit the time $(t < t_b)$ at which we wish to assess the damage. Thus,

$$A(t) = \int_0^{t/t_b} \frac{(\sigma_t - \sigma_{cr})^B}{(\sigma_{tM} - \sigma_{cr})^B} \, d\,(t/t_b) \qquad \text{(L-3)}$$

and

$$\Sigma D(t) = \frac{(\sigma_{tM} - \sigma_{cr})^{B} \; t_b A(t)}{P(n) \; (\sigma_{to} - \sigma_{cr})^{B} \; t_o a_T a_p} \qquad \text{(L-4)}$$

This relation is greatly simplified on dividing Equation (L-4) by (L-2) to give

$$\Sigma D(t) = A(t)/A \qquad \text{(L-5)}$$

Considering Equation (L-5), if the specimen which had been previously damaged fails at the time t, then the total damage on the specimen, by definition, must equal one. Since $\Sigma D(t)$ is less than one, the difference must be accounted for by the previous damage, ΣD_p. Thus, $\Sigma D(t)$ becomes

$$\Sigma D(t) = 1 - \Sigma D_p \qquad \text{(L-6)}$$

The values of A and A(t) can be obtained by integration of the data from a single uniaxial test record, from which the ratio A(t)/A and hence ΣD_p can be plotted versus testing time. From the original test record we can make a plot of the stress versus time and the strain versus the time. Using these three plots then for failure at any given testing time, t, corresponding values for the stress, the strain and ΣD_p can be read off. Plotting the values of σ and ε versus ΣD_p gives the desired failure curves and shows the reduction in the undamaged values ε_b and σ_b caused by the previous damage.

APPENDIX M

INPUT DATA FOR PRESSURIZATION TESTS ON A PBAN PROPELLANT

APPENDIX M

INPUT DATA FOR PRESSURIZATION TESTS ON A PBAN PROPELLANT

The data input procedures follow those shown in Appendix A. The Prony Series constants are tabulated in Table M-1. The time-temperature and the time-pressure shift factors are tabulated in Table M-2.

The cumulative damage parameters for this propellant are as follows:

$B = 8.75$

$\sigma_{to} = 71$ psi

$t_o = 1$ min.

For reference purposes we have provided a master relaxation curve, Figure M-1, and the curves for a_T and a_p, Figures M-2 and M-3, respectively.

TABLE M-1

MODULUS INPUT FOR PBAN PROPELLANT

log t/a_T, min.	E(T) psi	β_i min^{-1}	$A_i/3$	i
			8	0
-11	11,000	5×10^{10}	3241.4	1
-10	6,600	5×10^{8}	1187.7	2
-9	3,700	5×10^{6}	287.07	3
-8	1,850	5×10^{4}	108.14	4
-7	1,200	5×10^{2}	45.09	5
-6	750	5×10^{0}	32.153	6
-5	550	5×10^{-2}	18.209	7
-4	398	5×10^{-4}	14.289	8

TABLE M-2

SHIFT FACTORS FOR PBAN PROPELLANT

TIME-TEMPERATURE SHIFT FACTORS

Temp., °F	$\log_{10} a_T$
180	-2.508
150	-1.903
110	-0.876
77	0
40	2.097
0	4.057
-40	7.008
-75	10.528

TIME-PRESSURE SHIFT FACTOR

Press., psig.	a_p	$\log_{10} a_p$
0	1	0
200	51	1.71
600	150	2.176
1000	250	2.398

RELAXATION MODULUS FOR PBAN PROPELLANT

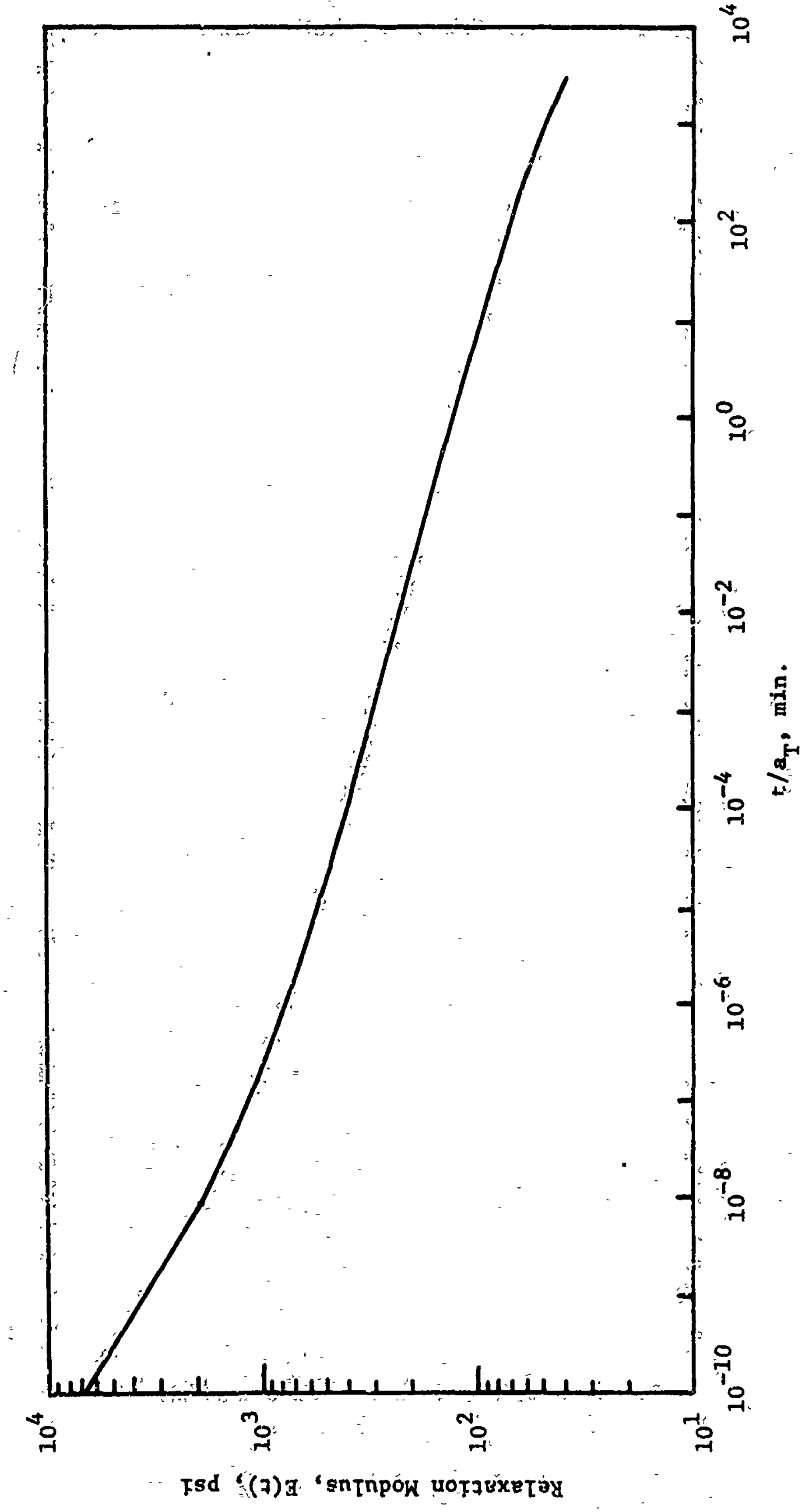

Figure M-1

TIME-TEMPERATURE SHIFT FACTOR FOR PBAN PROPELLANT

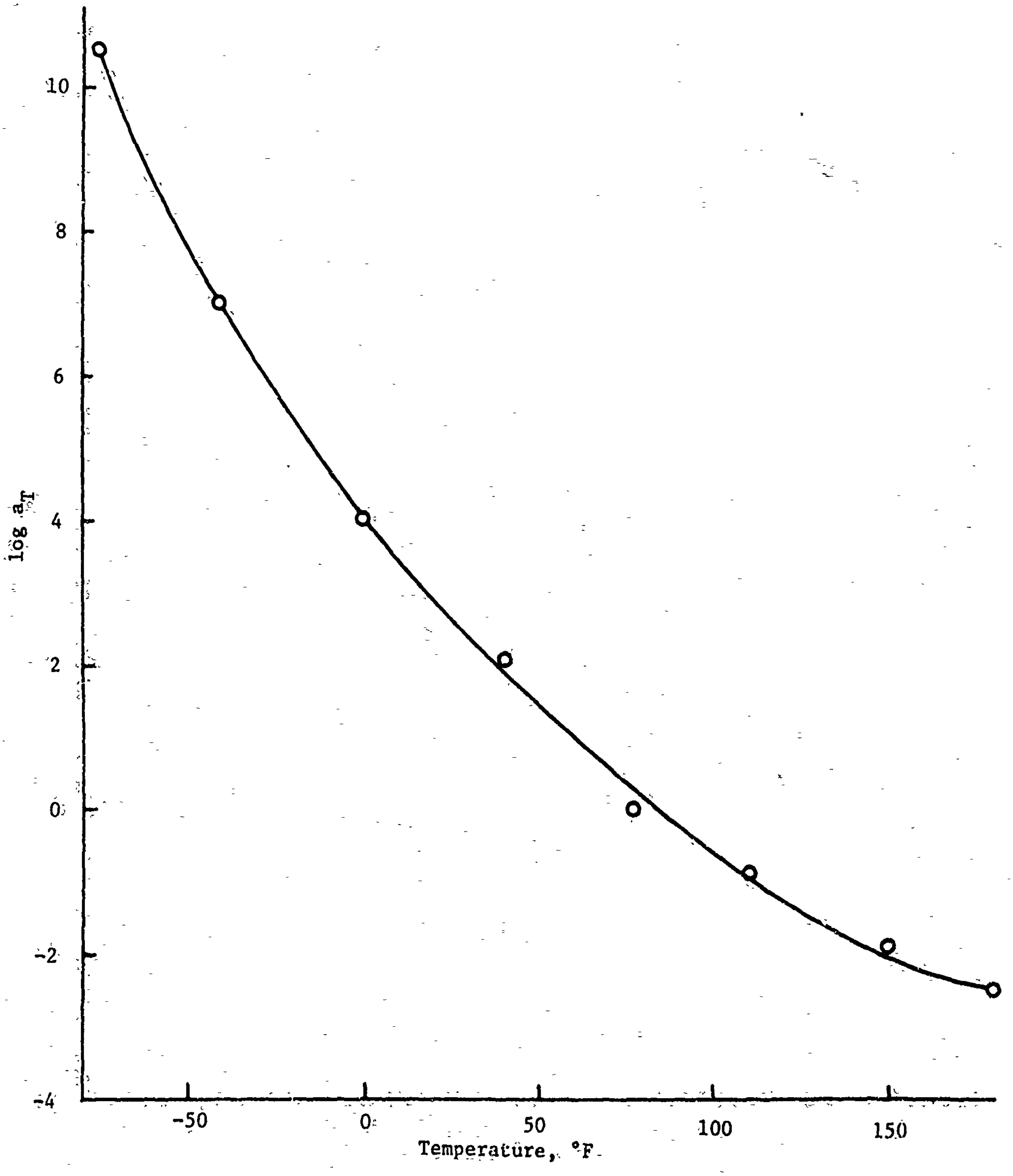

Figure M-2

TIME-PRESSURE SHIFT FACTOR FOR PBAN PROPELLANT

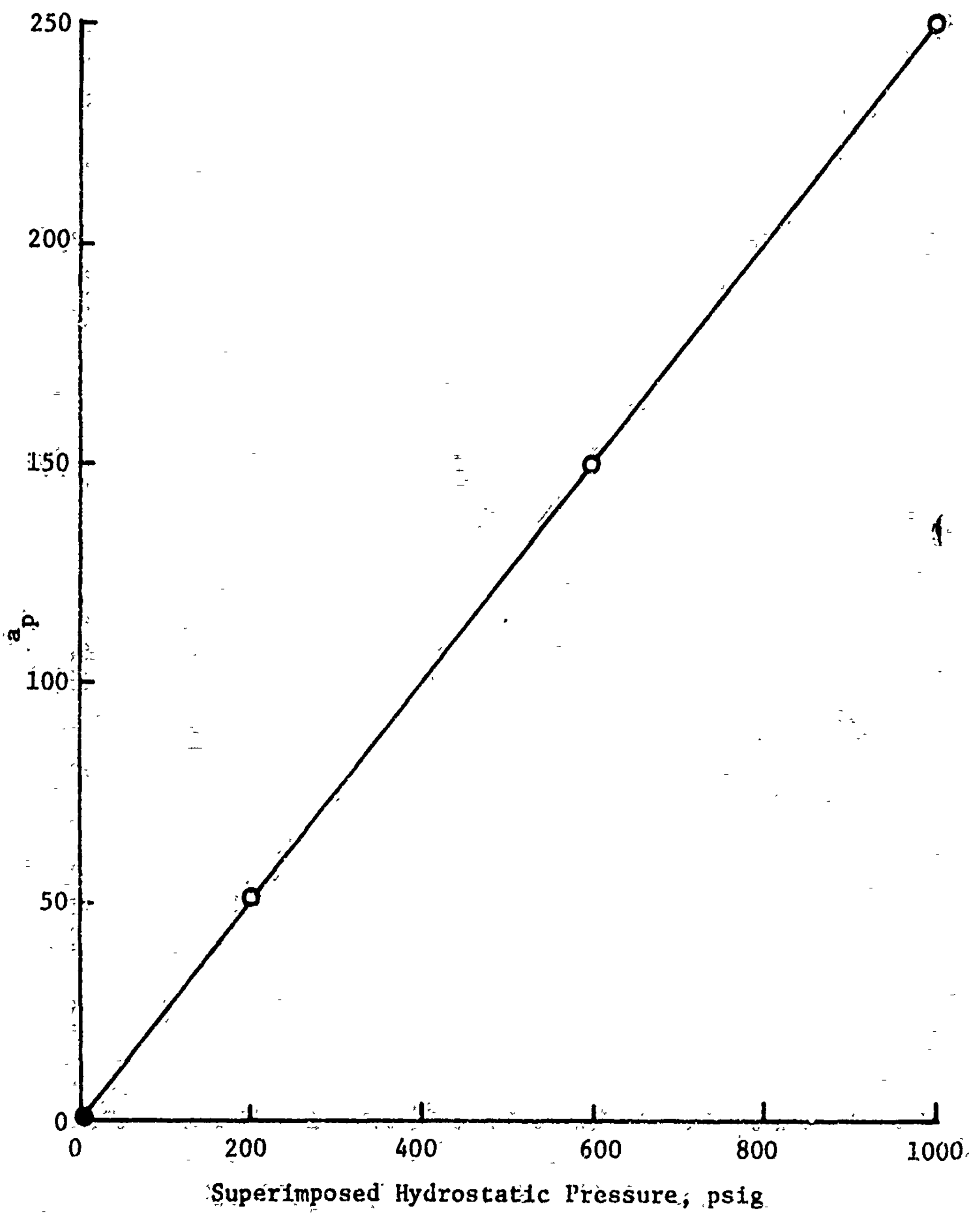

Figure M-3